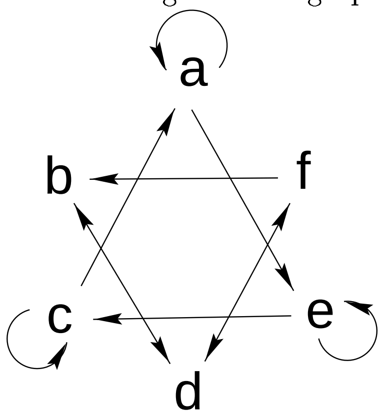
Determine the binary relation defined by the formula $F(x,y) = \exists u\,(rxu \wedge ruy)$ on the following directed graph:



	a	b	c	d	e	f
a	1		1		1	
b		1				1
c	1		1		1	
d		1		1		0
e	1		1		1	
f		1		1		1

Consider the following 5 sentences in the language of directed graphs:

F_1: $\forall x\,(rxx)$

F_2: $\exists x \forall y\,(rxy)$

F_3: $\forall x \forall y \forall z\,(rxy \wedge ryz \to rxz)$

F_4: $\forall x \forall y \big(rxy \to \exists z\,(rxz \wedge rzy)\big)$

F_5: $\forall x \exists y \forall z\,(rxy \wedge (rxz \to ryz))$

and the following two directed graphs:

$\mathbf{G}_1 = (G, r)$ where $G = \{0, 1\}$ and $r = \{(0,1), (1,1)\}$

$\mathbf{G}_2 = (N, |)$ where $N = \{0, 1, 2, \ldots\}$, the non-negative integers, with the usual 'divides' relation.

Find the truth values of each of the above sentences in each of the above structures, and enter these values (0 or 1) in the table below.

[This question will be marked as follows: each correct answer is worth 1 mark; each incorrect answer receives a penalty of -1; each blank receives 0 marks. However, the lowest possible total mark for this question is 0.]

	F_1	F_2	F_3	F_4	F_5
$\mathbf{G}_1$	0	0	1	1	1
$\mathbf{G}_2$	1	1	1	1	1

Put the following formula in prenex form:

$$\exists u\,(u < x) \to \forall y \forall z\,\big[(u + y < u + z) \to \exists w\,(w \cdot y < w \cdot z)\big]$$

Answer: $\forall v \forall y \forall z \exists w\,\Big((v < x) \to \big((u + y < u + z) \to (w \cdot y < w \cdot z)\big)\Big)$

Skolemize the following formula:

$$\exists x \forall y \forall z \exists w\,\big(x < y + z \;\to\; y + z < w\big)$$

Answer: $\forall y \forall z\,(c < y + z \to y + z < fyz)$

Convert the following argument into a set $\mathcal{C}$ of clauses such that the argument is valid iff the set $\mathcal{C}$ is not satisfiable.

$$\exists x\Big(\forall y\,(x \cdot y \le x) \;\vee \forall y \exists z\,\big((y \cdot z \le x) \wedge \neg\,(y \cdot x \le z)\big)\Big) \quad \therefore\; \forall x \exists y\,(x \cdot y \le y)$$

$$\exists x\Big(\forall y\,(x \cdot y \le x) \;\vee \forall w \exists z\,\big((w \cdot z \le x) \wedge \neg\,(w \cdot x \le z)\big)\Big), \quad \neg\,\forall x \exists y\,(x \cdot y \le y)$$

$$\exists x \forall w \exists z \forall y\Big((x \cdot y \le x) \;\vee \big((w \cdot z \le x) \wedge \neg\,(w \cdot x \le z)\big)\Big), \quad \exists x \forall y\,\neg\,(x \cdot y \le y)$$

Answer: $\{c \cdot y \le c,\; w \cdot fw \le c\}$, $\{c \cdot y \le c,\; \neg\,(w \cdot fw \le fw)\}$,

$\{\neg\,(d \cdot y \le y)\}$