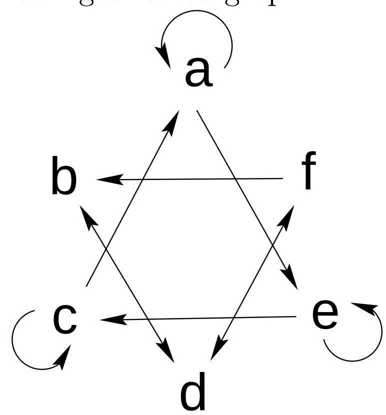
Assignment 8

Structures

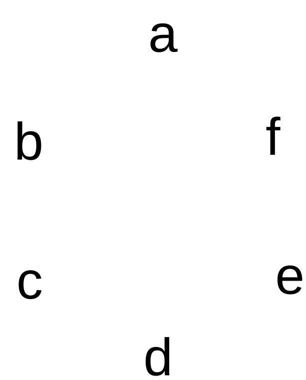
Fill in the table for the following directed graph:



	a	b	c	d	e	f
a						
b						
c						
d						
e						
f						

Draw the directed graph for the following table:

	a	b	c	d	e	f
a	1	1	0	0	0	0
b	0	1	1	0	0	0
c	0	0	1	1	0	0
d	0	0	0	1	1	0
e	0	0	0	0	1	1
f	0	0	0	0	0	1



Suppose that a first-order languge has only a constant symbol 0, a unary function symbol f, and a binary relation symbol r.

Then the third level S_3 of the Herbrand universe $\{0, f0, ff0, fff0\}$.

Determine the number of **distinct** ground clauses that one can make from the transitivity clause $\{\neg rxy,\ \neg ryz,\ rxz\}$ using S_3.

Answer: __________

Clauses

Let **S** be the structure on $\{0, 1, 2\}$ with a unary operation defined by $x' = 2x + 1$ (mod 3), and let $a \Box b$ mean that a and b have the same parity. Given the literals

$$L_1 \;=\; \neg\,(x' \Box y) \qquad L_2 \;=\; x \Box y'$$

determine all pairs (a, b) that satisfy the clause $C = \{L_1, L_2\}$ in **S** by filling in the following:

x	y	x'	y'	$x' \Box y$	$\neg\,(x' \Box y)$	$x \Box y'$	C
0	0						
0	1						
0	2						
1	0						
1	1						
1	2						
2	0						
2	1						
2	2						

Does **S** satisfy the clause C? ____________

Opp-unification

Suppose our first-order language has a binary relation symbol r and a binary operation symbol g. Show that the two clauses $\{rggxyzguv,\ rguzgugxy\}$ and $\{\neg\, rgxygyz\}$ are opp-unifiable, and find the most general opp-unifier (σ_1, σ_2).

$$\text{Thus}\,\sigma_1 = \begin{pmatrix} x \leftarrow & \qquad\qquad\qquad \\ y \leftarrow & \\ z \leftarrow & \\ u \leftarrow & \\ v \leftarrow & \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} x \leftarrow & \qquad\qquad\qquad \\ y \leftarrow & \\ z \leftarrow & \end{pmatrix}$$

Resolution Theorem Proving

For f, g unary function symbols and r a binary relation symbol fill in the reasons for the following resolution derivation:

1. $\{rfgxx\}$	given
2. $\{\neg rffgg0\}$	given
3. $\{\neg rxy,\ \neg ryz,\ rxz\}$	given
4. $\{\neg rxy,\ rfyfy\}$	given
5. $\{rffgxfx\}$	
6. $\{\neg rffgg0y,\ \neg ry0\}$	
7. $\{\neg rfg00\}$	
8. $\{\ \}$	

The following uses ground clauses from the given clauses of the previous problem. Identify the source of each ground clause (e.g., clause 3) that is an instance of one of the above given clauses, or otherwise justify the resolution step:

1. $\{rfg00\}$	
2. $\{rfgg0g0\}$	
3. $\{\neg rffgg0\}$	
4. $\{\neg rffgg0fg0,\ \neg rfg00,\ rffgg00\}$	
5. $\{\neg rfgg0g0,\ rffgg0fg0\}$	
6. $\{\neg rffgg0fg0,\ \neg rfg00\}$	
7. $\{rffgg0fg0\}$	
8. $\{\neg rfg00\}$	
9. $\{\ \}$	

For the language $\{\vee, \wedge, ', 0, 1\}$ of Boolean algebras write down the clauses $AX_{\equiv}$.

1. ____________________ Reflexive
2. ____________________ Symmetric
3. ____________________ Transitive
4. ____________________ Replacement for $\vee$
5. ____________________ Replacement for $\wedge$
6. ____________________ Replacement for $'$

Convert the following equational argument into a set of clauses such that the argument is valid iff the set of clauses is not satisfiable.

$$x \cdot y \approx u \cdot v \qquad \therefore\ x \cdot y \approx y \cdot x$$

And justify the derivation of the empty clause.

1. ____________________ Reflexive
2. ____________________ Symmetric
3. ____________________ Transitive
4. ____________________ Replacement for $\cdot$
5. ____________________ from the premiss
6. ____________________ from the conclusion
7. _________ $\{\ \}$ _________ ____________________