Assignment 7

Unification

In each of the following you are given a pair of terms that you are to test for being unifiable, and if they are unifiable, give the most general unifier.

(1) $x + (1 + (x + z))$ and $0 + (u + (v + (u + v)))$

(2) $x + (y \cdot (z + u))$ and $(u + (u \cdot v)) + ((v + u) \cdot u))$

(3) $u \cdot (x + (y \cdot (z + w))$ and $u \cdot ((u + (u \cdot v)) + ((v + u) \cdot u'))$

(4) $(x + (y + z)) + y$ and $u + (z + (v + v))$

For the Normal Form TRS given by

$$\mathcal{R} \approx \{fffx \longrightarrow ffx\}$$

find the normal forms for the following terms:

Term	Normal Form
fx	
ffy	
$fffz$	
$ffffu$	
$fffffv$	

For the Normal Form TRS given by

$$\mathcal{R} \approx \{fgx \longrightarrow gfx, \ ffx \longrightarrow fx\}$$

find the normal forms for the following terms:

Term	Normal Form
$gfgx$	
$gffgy$	
$gfgfz$	
$ffggu$	
$fgfgfv$	

For the Normal Form TRS given by

$$\mathcal{R} \approx \{(x + y) + z \longrightarrow x + (y + z)\}$$

find the normal forms for the following terms:

Term	Normal Form
$(x + x) + x$	
$(x + u) + (y + v)$	
$(x + (u + v)) + (v + u)$	
$((x + w) + (x + u)) + y$	
$(x + y) + ((y + z) + (z + w))$	

Indicate why the TRS

$$\mathcal{R} \approx \{x + y \longrightarrow x \cdot z\}$$

is not terminating for the term $x + y$ by filling in a few steps of

$x + y \longrightarrow_{\mathcal{R}}$ _____ $\longrightarrow_{\mathcal{R}}$ _____ $\longrightarrow_{\mathcal{R}}$ _____

Indicate why the TRS

$$\mathcal{R} \approx \{x + y \longrightarrow y + x\}$$

is not terminating for the term $x + y$ by filling in a few steps of

$x + y \longrightarrow_{\mathcal{R}}$ _____ $\longrightarrow_{\mathcal{R}}$ _____ $\longrightarrow_{\mathcal{R}}$ _____

Indicate why the terminating TRS

$$\mathcal{R} \approx \{fgx \longrightarrow fx, \; gfx \longrightarrow x\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$fgfx$	_____
$fgfx$	_____

Indicate why the terminating TRS

$$\mathcal{R} \approx \{x + (y + z) \longrightarrow z + x\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$x + (u + (v + w))$	_____
$x + (u + (v + w))$	_____

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Given the pair of term rewrite rules (with disjoint variables)

$$f\underline{gx} \longrightarrow fx \quad \text{and} \quad \underline{ggfu} \longrightarrow ggu$$

find the critical pair that results from unifying the underlined subterms:

Answer: _____

Given the pair of term rewrite rules (with disjoint variables)

$$\underline{fgffx} \longrightarrow fx \quad \text{and} \quad gg\underline{fu} \longrightarrow gfu$$

find the critical pair that results from unifying the underlined subterms:

Answer: _____

Given the pair of term rewrite rules (with disjoint variables)

$$x \cdot \underline{(y + z)} \longrightarrow (x \cdot y) + (x \cdot z) \quad \text{and} \quad \underline{(u + v) + w} \longrightarrow u + (v + w)$$

find the critical pair that results from unifying the underlined subterms:

Answer: _____