Name: ID:

Return to 10:30 a.m. / 1:30 p.m. class

PMath 330

Assignment 6

Mark _____

Substitution/Replacement

In each of the following inferences you are to choose the best answer for how the inference could be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1. $$\frac{x+y \approx y+x}{u+v \approx v+u}$$ _____

2. $$\frac{x\cdot(y+z) \approx (x\cdot y)+(x\cdot z)}{(x+y)\cdot z \approx (x\cdot z)+(y\cdot z)}$$ _____

3. $$\frac{x+y \approx y+x}{(x+y)+z \approx (y+x)+z}$$ _____

4. $$\frac{x\cdot y \approx u\cdot v}{(x\cdot y)\cdot w \approx (u\cdot v)\cdot w}$$ _____

5. $$\frac{x\cdot x \approx x}{(x\cdot x)\cdot(x\cdot x) \approx x\cdot x}$$ _____

Find a two element **counterexample** to the following equational argument:

$$\frac{x\cdot y \approx x \qquad (x+y)\cdot z \approx (x\cdot z)+(x\cdot y)}{x\cdot(y+z) \approx (x\cdot y)+(x\cdot z)}$$

$+$	a	b
a		
b		

$\cdot$	a	b
a		
b		

Find a three element **counterexample** to the following argument:

$$\frac{f^3(x) \approx x}{f^2(x) \approx x}$$

	f
a	
b	
c	

Fill in the reasons for the steps in the following derivation of $(y \cdot x) + (x \cdot y) \approx 0$ from the ring axioms R1–R9 plus the idempotent law $x \cdot x \approx x$:

R1.	$x + 0$	$\approx$	x	given
R2.	$x + (-x)$	$\approx$	0	given
R3.	$x + y$	$\approx$	$y + x$	given
R4.	$x + (y + z)$	$\approx$	$(x + y) + z$	given
R5.	$x \cdot 1$	$\approx$	x	given
R6.	$1 \cdot x$	$\approx$	x	given
R7.	$x \cdot (y \cdot z)$	$\approx$	$(x \cdot y) \cdot z$	given
R8.	$x \cdot (y + z)$	$\approx$	$(x \cdot y) + (x \cdot z)$	given
R9.	$(x + y) \cdot z$	$\approx$	$(x \cdot z) + (y \cdot z)$	given
10.	$x \cdot x$	$\approx$	x	given
11.	$(x + y) \cdot (x + y)$	$\approx$	$x + y$	
12.	$(x + y) \cdot (x + y)$	$\approx$	$((x + y) \cdot x) + ((x + y) \cdot y)$	
13.	$(x + y) \cdot x$	$\approx$	$(x \cdot x) + (y \cdot x)$	R9 Subs
14.	$(x \cdot x) + (y \cdot x)$	$\approx$	$x + (y \cdot x)$	10 Repl
15.	$(x + y) \cdot x$	$\approx$	$x + (y \cdot x)$	13, 14 Trans
16.	$((x + y) \cdot x) + ((x + y) \cdot y)$	$\approx$	$(x + (y \cdot x)) + ((x + y) \cdot y)$	
17.	$(x + y) \cdot y$	$\approx$	$(x \cdot y) + (y \cdot y)$	
18.	$y \cdot y$	$\approx$	y	
19.	$(x \cdot y) + (y \cdot y)$	$\approx$	$(x \cdot y) + y$	
20.	$(x + y) \cdot y$	$\approx$	$(x \cdot y) + y$	
21.	$(x + (y \cdot x)) + ((x + y) \cdot y)$	$\approx$	$(x + (y \cdot x)) + ((x \cdot y) + y)$	
22.	$((x + y) \cdot x) + ((x + y) \cdot y)$	$\approx$	$(x + (y \cdot x)) + ((x \cdot y) + y)$	
23.	$(x + y) \cdot (x + y)$	$\approx$	$(x + (y \cdot x)) + ((x \cdot y) + y)$	
24.	$(x + (y \cdot x)) + ((x \cdot y) + y)$	$\approx$	$(x + y) \cdot (x + y)$	23 Symm
25.	$(x + (y \cdot x)) + ((x \cdot y) + y)$	$\approx$	$x + y$	11, 24 Trans
26.	$(-x) + (x + (y \cdot x)) + ((x \cdot y) + y)$	$\approx$	$(-x) + (x + y)$	25 Repl
27.	$(-x) + x$	$\approx$	$x + (-x)$	
28.	$(-x) + x$	$\approx$	0	
29.	$(-x) + (x + y)$	$\approx$	$((-x) + x) + y$	

30. $((-x)+x)+y \approx 0+y$
31. $0+y \approx y+0$
32. $y+0 \approx y$
33. $0+y \approx y$
34. $((-x)+x)+y \approx y$
35. $(-x)+(x+y) \approx y$
36. $(-x)+\Big((x+(y\cdot x))+((x\cdot y)+y)\Big) \approx y$
37. $(-x)+\Big((x+(y\cdot x))+((x\cdot y)+y)\Big) \approx \Big((-x)+(x+(y\cdot x))\Big)+((x\cdot y)+y)$
38. $\Big((-x)+(x+(y\cdot x))\Big)+((x\cdot y)+y) \approx (-x)+\Big((x+(y\cdot x))+((x\cdot y)+y)\Big)$
39. $(-x)+(x+(y\cdot x)) \approx ((-x)+x)+(y\cdot x)$
40. $((-x)+x)+(y\cdot x) \approx 0+(y\cdot x)$
41. $0+(y\cdot x) \approx y\cdot x$
42. $((-x)+x)+(y\cdot x) \approx y\cdot x$
43. $(-x)+(x+(y\cdot x)) \approx y\cdot x$
44. $\Big((-x)+(x+(y\cdot x))\Big)+((x\cdot y)+y) \approx (y\cdot x)+((x\cdot y)+y)$
45. $(y\cdot x)+((x\cdot y)+y) \approx \Big((-x)+(x+(y\cdot x))\Big)+((x\cdot y)+y)$
46. $(y\cdot x)+((x\cdot y)+y) \approx (-x)+\Big((x+(y\cdot x))+((x\cdot y)+y)\Big)$
47. $(y\cdot x)+((x\cdot y)+y) \approx y$
48. $\Big((y\cdot x)+((x\cdot y)+y)\Big)+(-y) \approx y+(-y)$
49. $y+(-y) \approx 0$
50. $\Big(((y\cdot x)+(x\cdot y))+y\Big)+(-y) \approx 0$
51. $((y\cdot x)+(x\cdot y))+(y+(-y)) \approx \Big(((y\cdot x)+(x\cdot y))+y\Big)+(-y)$
52. $((y\cdot x)+(x\cdot y))+(y+(-y)) \approx 0$
53. $((y\cdot x)+(x\cdot y))+(y+(-y)) \approx ((y\cdot x)+(x\cdot y))+0$
54. $((y\cdot x)+(x\cdot y))+0 \approx (y\cdot x)+(x\cdot y)$
55. $((y\cdot x)+(x\cdot y))+(y+(-y)) \approx (y\cdot x)+(x\cdot y)$
56. $(y\cdot x)+(x\cdot y) \approx ((y\cdot x)+(x\cdot y))+(y+(-y))$
57. $(y\cdot x)+(x\cdot y) \approx 0$