Using Critical Pairs in Equational Theorem Proving

We have the following information from an equational proof provided by S.O.B.B.:

88	$x + ((x+y)\cdot z)$	$=$	$(x+y)\cdot(x+z)$	
127	$x\cdot(y\cdot(z+x))$	$=$	$y\cdot x$	
350	$x+(y\cdot(x+z))$	$=$	$x+(y\cdot z)$	
628	$x\cdot(y+(z\cdot(u+x)))$	$=$	$x\cdot(y+z)$	
734	$x+(y\cdot z)$	$=$	$(x+z)\cdot(x+y)$	[para_into, 88.1.1.2, 127.1.1, demod, 350, 628]

Let us fill in the details of how step 734 is derived. **Paramodulation**, when applied to equations, means that one is to form an equation using the critical pair obtained from the two specified equations that follow. In our case, the equations are 88 and 127. The subterms of the two sides of an equation are labelled by taking the tree for the equation, starting with = at the top, labelling the top node 1, and then labelling successive nodes by 1 for leftmost branch, 2 for next branch, etc. Thus 88.1.1.2 specifies the subterm $(x+y)\cdot z$ of the left side of 88, and 127.1.1 specifies the entire left side of 127. So we want to find the critical pair equation corresponding to these subterms:

$$\begin{array}{lrcl} 88 & x+\boxed{((x+y)\cdot z)} & = & (x+y)\cdot(x+z) \\ 127 & \boxed{x\cdot(y\cdot(z+x))} & = & y\cdot x \end{array}$$

The first thing to do is to make the variables disjoint:

$$\begin{array}{lrcl} 88 & x+\boxed{((x+y)\cdot z)} & = & (x+y)\cdot(x+z) \\ 127 & \boxed{X\cdot(Y\cdot(Z+X))} & = & Y\cdot X \end{array}$$

The most general unifier of the boxed terms is

$$\mu = \begin{pmatrix} X \leftarrow x+y \\ z \leftarrow Y\cdot(Z+(x+y)) \end{pmatrix}$$

Applying this to 88 and 127 gives

$$\begin{array}{lrcl} 88\mu & x+\boxed{(x+y)\cdot(Y\cdot(Z+(x+y)))} & = & (x+y)\cdot(x+(Y\cdot(Z+(x+y)))) \\ 127\mu & \boxed{(x+y)\cdot(Y\cdot(Z+(x+y)))} & = & Y\cdot(x+y) \end{array}$$

From this we have the Critical Pair Equation:

$$(**) \qquad x+(Y\cdot(x+y)) \;=\; (x+y)\cdot(x+(Y\cdot(Z+(x+y))))$$

Demodulation means to apply the specified equation like a term rewrite. By applying 350 to the left side of (**), and 628 to the right side, we obtain

$$x+(Y\cdot y) \;=\; (x+y)\cdot(x+Y).$$

Changing the variables gives

$$x+(y\cdot z) \;=\; (x+z)\cdot(x+y).$$