# A Primer on Sets, Classes

Set (or Class)

We take the notion of "A is a set" or "A is a class" as understood—no definition is given.

Intuitively a set is any collection of things—the things in the set are called its **elements** or **members**.

We use the words "set" and "class" interchangeably, just as they did before 1900.

According to Hilbert and Ackermann, 1928, philosophers prefer the word "class" and mathematicians prefer the word "set".

We like to draw sets as regions in the plane, and indicate elements by dots.
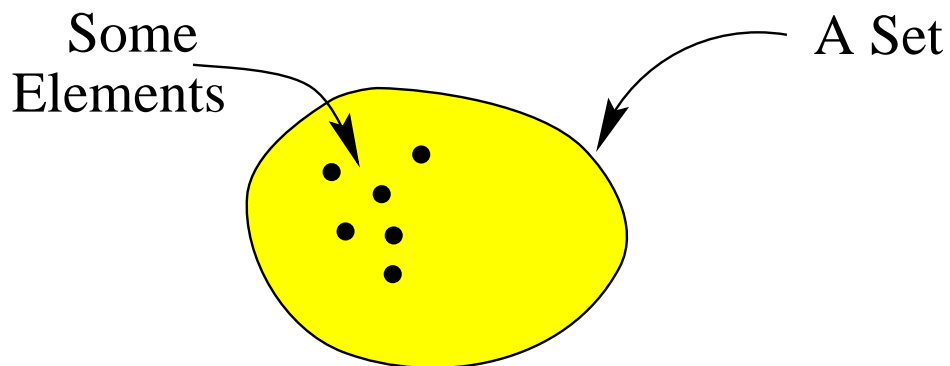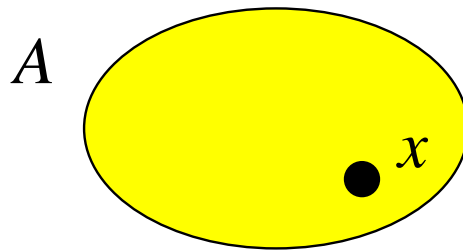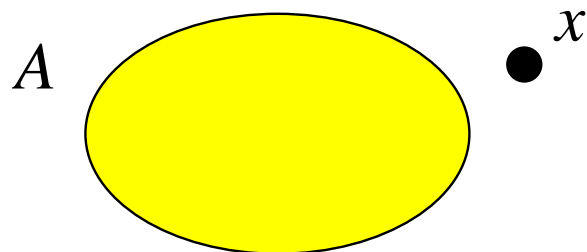
Set Membership

We take the notion of set membership

$$x \in A \quad \boxed{\text{Read: } x \ \textbf{is an element of} \ A}$$

as understood—no definition is given.

A x

The notation $x \notin A$ means "x **is not an element of** A".

A x

Equality of Sets

Two sets A and B are **equal** iff they have precisely the same elements,

that is, iff every member of A is a member of B, and vice-versa.

The Empty Set

We define the set

$$\emptyset \quad \boxed{\text{Read: } \textbf{empty set}}$$

to be a set with no elements in it.

Exercise: Explain why there can be only one empty set.

Intension and Extension

The notion of a *rose* has both **intension** and **extension**.

Intension has to do with **properties**:

$$\boxed{x \text{ is a rose.}}$$

Extension has to do with **sets**:

$$\boxed{\text{The set of all roses.}}$$

Properties are **true** or **false** for objects.

Sets are just sets.

Using Properties to Make Sets

We can (**naively**) express the construction of a **set** using a **property** with **set-builder notation**:

$$\{x : \mathcal{P}(x)\}$$

Read: the set of elements for which $\mathcal{P}$ holds

For example:

$$\text{roses} = \{x : x \text{ is a rose}\}.$$

In advanced set theory one has to be careful about the properties that one ascribes to sets.

This is because of the famous **Russell Paradox** discovered in 1901 by Bertrand Russell.

In such advanced work one likes to treat sets as special kinds of "not too big" classes.

However such problems do not arise in our work.

Russell's Paradox (1901)

Let $\mathcal{P}(x)$ be the property $x \notin x$.

Let $A = \{x : x \notin x\}$.

Then $A \in A$ iff $\mathcal{P}(A)$ iff $A \notin A$.

But this is a contradiction.

We have to be more careful about how we build sets!

Using Properties to Make Sets

Given any property $\mathcal{P}(x)$ **and** set A there is a set whose elements are the elements x of A for which $\mathcal{P}(x)$ is true.

The usual way of describing this set is by set-builder notation, namely

$$\{x \in A : \mathcal{P}(x)\}$$

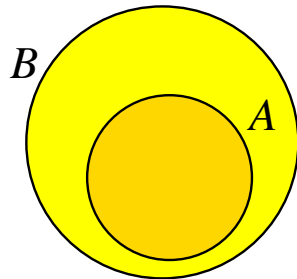Read: the set of elements in A for which $\mathcal{P}$ holds

For example: $\{x \in N : x \text{ is even.}\}$.

Subset

Given any two sets A and B we define

$A \subseteq B$ | Read: A **is a subset of** B |

by: every element of A is an element of B .

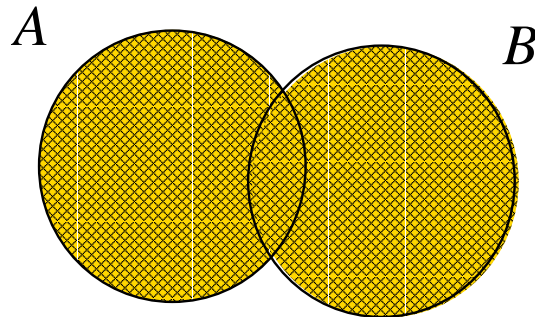(An **Euler Diagram**)

The notation $A \nsubseteq B$ means

A **is not a subset of** B.

Union

Given two sets A and B we define

$$A \cup B \quad \boxed{\text{Read:} \quad A \text{ \textbf{union} } B}$$

by $x \in A \cup B$ iff $x \in A$ or $x \in B$.

The elements of $A \cup B$ are in the shaded region.

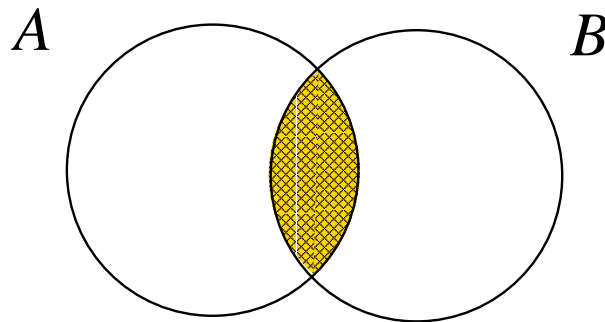**In the textbook we use a different convention**, namely we shade the regions that are known to be **empty**.

Intersection

Given two sets A and B we define

$A \cap B$ │ Read: A **intersection** B │

by

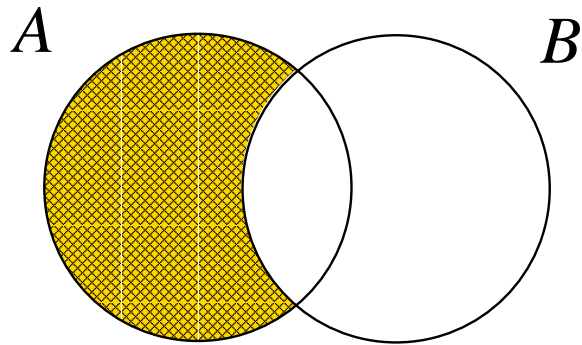$x \in A \cap B$ iff $x \in A$ and $x \in B$.

Set Difference

Given two sets A and B we define

$$A \setminus B \quad \boxed{\text{Read: } A \text{ minus } B}$$

by

$$x \in A \setminus B \quad \text{iff} \quad x \in A \text{ and } x \notin B.$$

The Universe of Discourse

In a given context it is understood that the sets being considered are all subsets of a given set U called the **universe (of discourse)**.

Complement

Given A (a subset of U) we define

A' | Read: **the complement of A**

by $x \in A'$ iff $x \notin A$ (assuming $x \in U$).