1.	4.
2.	5.
3.	6.

Total = **/90**

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PMath 330 MIDTERM Friday, October 27, 2000

There are 4 pages, with 6 problems.

Problem 1

12 marks

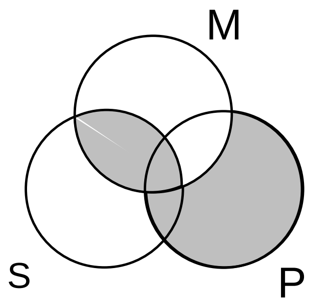
Give the **mood** and **figure** of the syllogism to the right, and fill in its Venn diagram (using Modern Standards, as in the text). State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

No M is S
All P is M
Some S is not P.

Mood: AEO **Figure:** 4

Valid (Modern): NO

Valid (Aristotelian): YES



Problem 2

13 marks

Apply unit resolution to determine if the given set of clauses is satisfiable:

1.	$\{P\}$	given	6.	$\{Q, \neg R\}$	$1, 3$
2.	$\{P, \neg Q, \neg R\}$	given	7.	$\{\neg Q, R\}$	$1, 5$
3.	$\{\neg P, Q, \neg R\}$	given	8.	$\{P, \neg R\}$	$2, 4$
4.	$\{Q\}$	given	9.	$\{\neg P, R\}$	$4, 5$
5.	$\{\neg P, \neg Q, R\}$	given	10.	$\{R\}$	$1, 9$
			11.	$\{P, \neg Q\}$	$2, 10$
			12.	$\{\neg P, Q\}$	$3, 10$

ANSWER: (Reasons) YES. Cannot derive the empty clause.

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Problem 3

10 marks

Given the following combined truth table for five formulas

	P	Q	R	F_1	F_2	F_3	F_4	F_5			
1.	1	1	1	0	1	1	1	0			
2.	1	1	0	1	0	1	1	1			
3.	1	0	1	0	1	0	1	1			
4.	1	0	0	1	1	0	1	0			
5.	0	1	1	1	1	1	1	0			
6.	0	1	0	1	0	1	1	1			
7.	0	0	1	0	0	0	0	1			
8.	0	0	0	1	0	0	0	0			

(a) What is the Conjunctive Normal Form of $F_2 \to F_3$?

$$(\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R)$$

(b) Is the set $\{F_1 \to F_2, F_2 \to F_3, F_3 \to F_1\}$ satisfiable? (Why?)

YES, by line 7.

(c) Is the argument F_2, F_3, F_4, F_5 $\therefore$ F_1 valid? (Why?)

YES, as the premisses are not satisfiable.

Problem 4

15 marks

Given that $F(X) = AX \cup BX'$, where A and B are subsets of a given universe U, prove: there is an $X \subseteq U$ such that $F(X) = 0$ if and only if $AB = 0$.

PROOF:

First let us suppose that $AX \cup BX' = 0$, for some X. Then one has $AX = 0$ and $BX' = 0$. Thus $A \subseteq X'$ and $B \subseteq X$. But then $AB \subseteq X'X = 0$, so $AB = 0$. This proves one direction.

Next let us suppose that $AB = 0$. Then $F(B) = AB \cup BB' = 0$.

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Problem 5

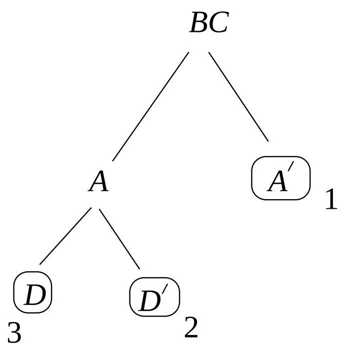
20 marks

You are given the premisses $A'B = 0$, $CD' = 0$, and $AD = 0$.

(a) Use Lewis Carroll's tree method to show that $BC = 0$ is a valid conclusion.

1. $A'B = 0$
2. $CD' = 0$
3. $AD = 0$

$BC = 0$



(b) Use Boole's elimination method to show that $BC = 0$ is the most general conclusion for the classes B and C.

SOLUTION:

The premisses are equivalent to the single equation $E(A, B, C, D) = 0$ where

$$E(A, B, C, D) = A'B \cup CD' \cup AD.$$

The most general conclusion for the classes B and C is $F(B, C) = 0$ where

$$\begin{aligned} F(B, C) &= E(1, B, C, 1)E(1, B, C, 0)E(0, B, C, 1)E(0, B, C, 0) \\ &= (1)(C)(B)(B \cup C) \\ &= BC. \end{aligned}$$

Thus $BC = 0$ is the most general conclusion for the classes B and C.

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Problem 6

20 marks

(a) Assuming just that the set of standard connectives (including the standard constants) is adequate, show that $\mathcal{C} = \{\to,\ 0\}$ is adequate.

SOLUTION:

$$\begin{aligned}
1 &\sim 0 \to 0 \\
\neg P &\sim P \to 0 \\
P \vee Q &\sim (P \to 0) \to Q \\
P \wedge Q &\sim ((P \to (Q \to 0)) \to 0) \\
P \leftrightarrow Q &\sim (P \to (Q \to 0)) \to (((P \to 0) \to Q) \to 0)
\end{aligned}$$

(b) Give the truth table for the Sheffer stroke $|$. Then use the fact that $\mathcal{C}$ in (a) is adequate to show that the Sheffer stroke is adequate.

SOLUTION:

P	Q	$P \vert Q$
1	1	0
1	0	1
0	1	1
0	0	1

Since

$$\begin{aligned}
0 &\sim (P|(P|P))|(P|(P|P)) \\
P \to Q &\sim P|(P|Q)
\end{aligned}$$

and $\mathcal{C}$ is adequate, it follows that the Sheffer stroke is adequate.