PMath 330 §02 **(10:30 a.m.)** MIDTERM Friday, October 29, 1999

There are 4 pages, with 6 problems.

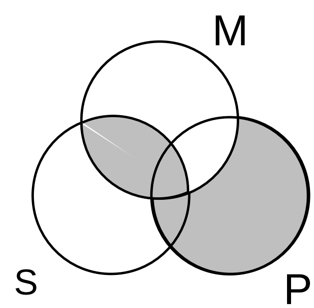
Problem 1

12 marks

Give the **mood** and **figure** of the syllogism to the right, and fill in the Venn diagram [use more than one if needed] to determine if the syllogism is indeed valid. State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

All M is S
No M is P

Some S is not P.



Mood: EAO **Figure:** 3

Valid (Modern): NO

Valid (Aristotelian): YES

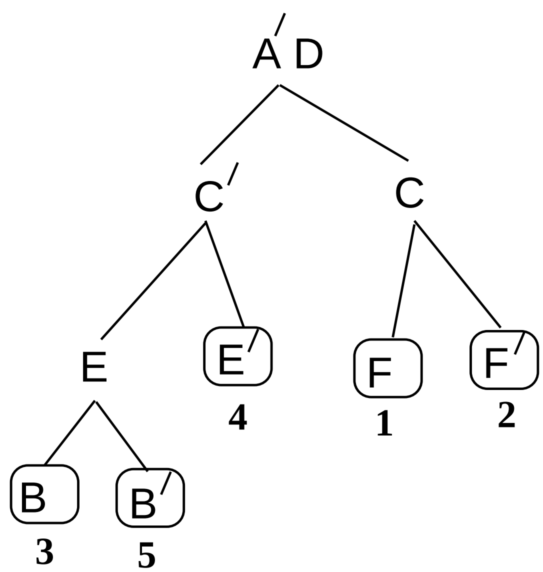
Problem 2

13 marks

Show that the following argument is valid by filling in the Lewis Carroll tree on the right, including appropriate numbers for the boxes.

1. $CDF = 0$
2. $A'CF' = 0$
3. $BDE = 0$
4. $A'C'E' = 0$
5. $A'B'C' = 0$

$A'D = 0$



Problem 3

10 marks

Given the following five formulas and their combined truth table

$F_1 : \neg R \vee Q$
$F_2 : R \vee ((P \rightarrow R) \leftrightarrow Q)$
$F_3 : \neg Q \rightarrow Q$
$F_4 : (Q \leftrightarrow P) \rightarrow (Q \wedge (P \leftrightarrow Q))$
$F_5 : \neg R \leftrightarrow Q$

	P	Q	R	F_1	F_2	F_3	F_4	F_5	$F_1 \rightarrow F_5$		
1.	1	1	1	1	1	1	1	0	0		
2.	1	1	0	1	0	1	1	1	1		
3.	1	0	1	0	1	0	1	1	1		
4.	1	0	0	1	1	0	1	0	0		
5.	0	1	1	1	1	1	1	0	0		
6.	0	1	0	1	1	1	1	1	1		
7.	0	0	1	0	1	0	0	1	1		
8.	0	0	0	1	0	0	0	0	0		

(a) What is the Conjunctive Normal Form of $F_1 \rightarrow F_5$?

$$(\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee R)$$

(b) Is the set $\{F_1, \neg F_2, F_3\}$ satisfiable? (Why?)

YES (Row 3)

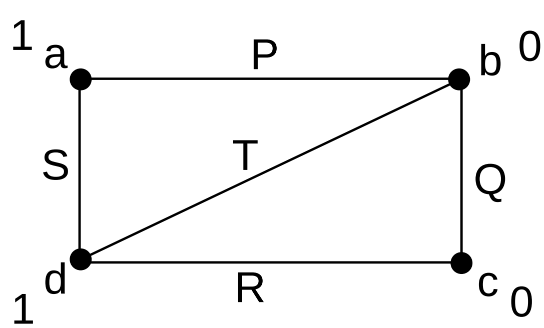
(c) Is the argument $\neg F_1,\ F_3 \therefore F_4$ valid? (Why?)

YES (The premisses are not satisfied in any row.)

Problem 4

14 marks

Find the graph clauses for the labelled graph on the right, and state (with reasons) whether or not this set of clauses is satisfiable.



$Clauses(a)$:	$\{P, S\}$	$\{\neg P, \neg S\}$		
$Clauses(b)$:	$\{\neg P, Q, T\}$	$\{P, \neg Q, T\}$	$\{P, Q, \neg T\}$	$\{\neg P, \neg Q, \neg T\}$
$Clauses(c)$:	$\{\neg Q, R\}$	$\{Q, \neg R\}$		
$Clauses(d)$:	$\{R, S, T\}$	$\{\neg R, \neg S, T\}$	$\{\neg R, S, \neg T\}$	$\{R, \neg S, \neg T\}$

As the TOTAL CHARGE is 0, this collection of 12 clauses **is satisfiable**.

Problem 5

10 marks

The following gives the first part of the completeness proof for the FŁ proof system. Fill in the reasons to justify Lemma D.0.5.

> The Frege-Łukasiewicz Propositional Logic
>
> **Propositional Variables:** $P, Q, \ldots$
>
> **Connectives:** $\neg, \rightarrow$
>
> **Rule of inference:** (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
>
> **Axiom schemata:**
>
> **A1:** $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
>
> **A2:** $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
>
> **A3:** $(\neg \mathsf{F} \rightarrow \neg \mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

Lemma A: If F is an axiom then $\vdash \mathsf{F}$.

Lemma B: If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

Lemma D.0.5: $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

Proof. The following gives a derivation of $\mathsf{F} \rightarrow \mathsf{F}$:

1.	$\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})$	$A1$
2.	$(\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})) \rightarrow ((\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F}))$	$A2$
3.	$(\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$	$MP\ 1, 2$
4.	$\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$	$A1$
5.	$\mathsf{F} \rightarrow \mathsf{F}$.	$MP\ 3, 4$

Problem 6

16 marks

(a) Without assuming any set of connectives is adequate, show that $\mathcal{C} = \{\wedge,\ \neg\}$ is adequate.

[Blackboard Note: Assume the standard connectives are adequate.]

$$\begin{array}{rcl} 0 & \sim & P \wedge \neg P \\ 1 & \sim & \neg(P \wedge \neg P) \\ P \vee Q & \sim & \neg(\neg P \wedge \neg Q) \\ P \rightarrow Q & \sim & \neg(P \wedge \neg Q) \\ P \leftrightarrow Q & \sim & \neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q) \\ \hline \neg P & \sim & \neg P \\ P \wedge Q & \sim & P \wedge Q \end{array}$$

(b) Use (a) to show that the Sheffer stroke $|$ is adequate.

$$\begin{array}{rcl} \neg P & \sim & P|P \\ P \wedge Q & \sim & (P|Q)|(P|Q) \end{array}$$