Mark = ______/60 Name ____________________
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PM 330 MIDTERM October 30, 1998

There are 4 pages, with 4 problems.

Problem 1

16 marks

Translate the following argument (of Lewis Carroll) into equations in the calculus of classes and use Lewis Carroll's tree method to show that the argument is valid.

1. Animals, that do not kick, are always unexcitable.
2. Donkeys have no horns.
3. A buffalo can always toss one over a gate.
4. No animals that kick are easy to swallow.
5. No hornless animals can toss one over a gate.
6. All animals are excitable, except buffaloes.

   Therefore, donkeys are not easy to swallow.

The universe of discourse is "animals", and the classes to be used are:

T: animals that are able to toss one over a gate
B: buffaloes
D: donkeys
S: easy to swallow animals
E: excitable animals
H: horned animals
K: kicking animals

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Problem 2

14 marks

Simplify the following propositional formula.

Do this any way you can, but be sure to show enough work so the marker can tell how you got the answer. If you use the fundamental equivalences you do **not** *need to name them, or say if you are using replacement, substitution, etc.*

$$((R \vee \neg P \vee \neg Q) \leftrightarrow (P \wedge Q \rightarrow R)) \rightarrow ((\neg S \wedge T) \vee \neg S) \wedge (T \leftrightarrow (\neg T \leftrightarrow S))$$

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Problem 3

Prove by **induction on formulas** that the set of connectives $\{\wedge, \leftrightarrow\}$ is not adequate.

16 marks

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Problem 4

14 marks

The following gives the first part of the completeness proof for the FŁ proof system. Fill in the reasons to justify Lemma D.0.12.

> The Frege-Łukasiewicz Propositional Logic
> **Propositional Variables:** $P, Q, \ldots$
> **Connectives:** $\neg, \rightarrow$
> **Rule of inference:** (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
> **Axiom schemata:**
> **A1:** $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
> **A2:** $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
> **A3:** $(\neg \mathsf{F} \rightarrow \neg \mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

Lemma A If F is an axiom then $\vdash \mathsf{F}$.

Lemma B If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

Lemma D.0.5 $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

Lemma D.0.6 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{G}$.

Lemma D.0.7 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \subseteq \mathcal{S}'$, then $\mathcal{S}' \vdash \mathsf{F}$.

Lemma D.0.8 $\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G}$ iff $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$.

Lemma D.0.9 If $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$ and $\mathcal{S} \vdash \mathsf{G} \rightarrow \mathsf{H}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Lemma D.0.10 If $\mathcal{S} \vdash \mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})$ and $\mathcal{S} \vdash \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Lemma D.0.11 $\vdash \neg \mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{G})$.

Lemma D.0.12 $\vdash \neg\neg \mathsf{F} \rightarrow \mathsf{F}$.

Proof

1.		$\vdash$	$\neg\neg \mathsf{F} \rightarrow (\neg \mathsf{F} \rightarrow \neg\neg\neg \mathsf{F})$	⋆
2.		$\vdash$	$(\neg \mathsf{F} \rightarrow \neg\neg\neg \mathsf{F}) \rightarrow (\neg\neg \mathsf{F} \rightarrow \mathsf{F})$	⋆
3.		$\vdash$	$\neg\neg \mathsf{F} \rightarrow (\neg\neg \mathsf{F} \rightarrow \mathsf{F})$	⋆
4.	$\neg\neg \mathsf{F}$	$\vdash$	$\neg\neg \mathsf{F} \rightarrow \mathsf{F}$	⋆
5.	$\neg\neg \mathsf{F}$	$\vdash$	$\neg\neg \mathsf{F}$	⋆
6.	$\neg\neg \mathsf{F}$	$\vdash$	F	⋆
7.		$\vdash$	$\neg\neg \mathsf{F} \rightarrow \mathsf{F}.$	⋆