1.	4.
2.	5.
3.	6.

Total = /75

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PMath 330 §01 **(1:30 p.m.)** MIDTERM Friday, October 29, 1999

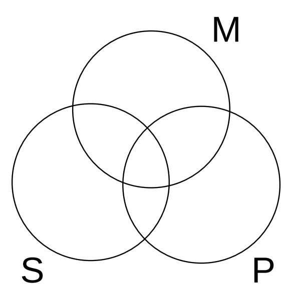
There are 4 pages, with 6 problems.

Problem 1

12 marks

Give the **mood** and **figure** of the syllogism to the right, and fill in the Venn diagram [use more than one if needed] to determine if the syllogism is indeed valid. State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

All S is M
No P is M
____
Some S is not P.



Mood:____ **Figure:**____

Valid (Modern):____

Valid (Aristotelian):____

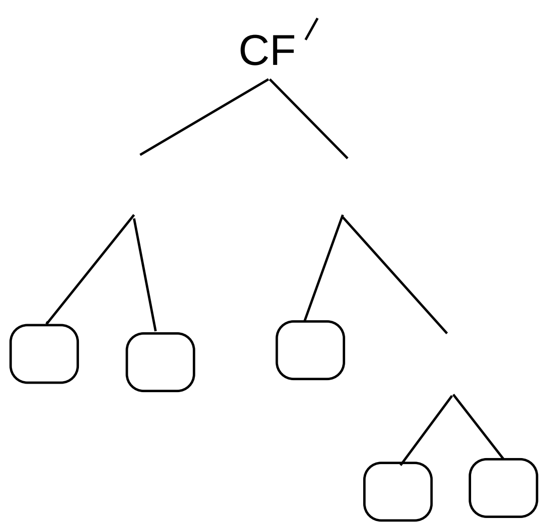
Problem 2

13 marks

Show that the following argument is valid by filling in the Lewis Carroll tree on the right, including appropriate numbers for the boxes.

1. $ACD = 0$
2. $A'DF' = 0$
3. $BCE = 0$
4. $B'D'F' = 0$
5. $D'E'F' = 0$

$$\overline{}$$
$CF' = 0$



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Problem 3

10 marks

Given the following five formulas and their combined truth table

$F_1 : \neg R \vee Q$

$F_2 : R \vee ((P \to R) \leftrightarrow Q)$

$F_3 : \neg Q \to Q$

$F_4 : (Q \leftrightarrow P) \to (Q \wedge (P \leftrightarrow Q))$

$F_5 : \neg R \leftrightarrow Q$

	P	Q	R	F_1	F_2	F_3	F_4	F_5		
1.	1	1	1	1	1	1	1	0		
2.	1	1	0	1	0	1	1	1		
3.	1	0	1	0	1	0	1	1		
4.	1	0	0	1	1	0	1	0		
5.	0	1	1	1	1	1	1	0		
6.	0	1	0	1	1	1	1	1		
7.	0	0	1	0	1	0	0	1		
8.	0	0	0	1	0	0	0	0		

(a) What is the Conjunctive Normal Form of $F_2 \to F_5$?

(b) Is the set $\{\neg F_1, F_2, \neg F_3\}$ satisfiable? (Why?)

(c) Is the argument F_1, $\neg F_3 \therefore F_4$ valid? (Why?)

Problem 4

12 marks

State the **pigeonhole principle** $\mathbb{P}_2$ (this means there are two pigeonholes) and express it as a collection of clauses. (This was the last topic covered in the lectures on clauses.)

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Problem 5

12 marks

Fill in the reasons to justify Lemma D.0.10.

> The Frege-Łukasiewicz Propositional Logic
>
> **Propositional Variables:** $P, Q, \ldots$
>
> **Connectives:** $\neg, \rightarrow$
>
> **Rule of inference:** (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
>
> **Axiom schemata:**
>
> **A1:** $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
>
> **A2:** $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
>
> **A3:** $(\neg \mathsf{F} \rightarrow \neg \mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

Lemma A If F is an axiom then $\vdash \mathsf{F}$.

Lemma B If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

Lemma D.0.5 $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

Lemma D.0.6 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{G}$.

Lemma D.0.7 If $\mathcal{S} \vdash \mathsf{F}$ and $\mathcal{S} \subseteq \mathcal{S}'$, then $\mathcal{S}' \vdash \mathsf{F}$.

Lemma D.0.8 $\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G}$ iff $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$.

Lemma D.0.9 If $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{G}$ and $\mathcal{S} \vdash \mathsf{G} \rightarrow \mathsf{H}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Lemma D.0.10 If $\mathcal{S} \vdash \mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})$ and $\mathcal{S} \vdash \mathsf{G}$, then $\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}$.

Proof:

1.	$\mathcal{S} \vdash \mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})$	______________________
2.	$\mathcal{S} \vdash \mathsf{G}$	______________________
3.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G}$	______________________
4.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{G} \rightarrow \mathsf{H}$	______________________
5.	$\mathcal{S} \cup \{\mathsf{F}\} \vdash \mathsf{H}$	______________________
6.	$\mathcal{S} \vdash \mathsf{F} \rightarrow \mathsf{H}.$	______________________

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Problem 6

16 marks

(a) Without assuming any set of connectives is adequate, show that $\mathcal{C} = \{\rightarrow, \neg\}$ is adequate.

(b) Use (a) to show that the Schröder connective $\curlywedge$ is adequate.