1.	4.
2.	5.
3.	6.

Total = /75

Name ____________________

ID ____________________

PMath 330 §02 **(10:30 a.m.)** MIDTERM Friday, October 29, 1999

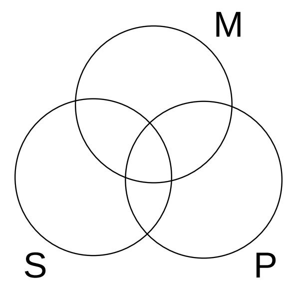
There are 4 pages, with 6 problems.

Problem 1

12 marks

Give the **mood** and **figure** of the syllogism to the right, and fill in the Venn diagram [use more than one if needed] to determine if the syllogism is indeed valid. State whether the syllogism is valid under **Modern Standards** as well as under **Aristotelian Standards**.

All M is S
No M is P
____
Some S is not P.



Mood:_____ **Figure:**_____

Valid (Modern):_____

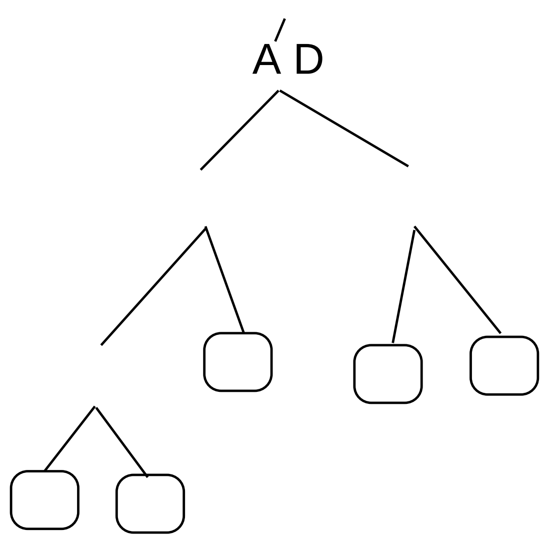
Valid (Aristotelian):_____

Problem 2

13 marks

Show that the following argument is valid by filling in the Lewis Carroll tree on the right, including appropriate numbers for the boxes.

$$
\begin{array}{rl}
1. & CDF = 0 \\
2. & A'CF' = 0 \\
3. & BDE = 0 \\
4. & A'C'E' = 0 \\
5. & A'B'C' = 0 \\
\hline
 & A'D = 0
\end{array}
$$



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Problem 3

10 marks

Given the following five formulas and their combined truth table

$F_1 : \neg R \lor Q$

$F_2 : R \lor ((P \to R) \leftrightarrow Q)$

$F_3 : \neg Q \to Q$

$F_4 : (Q \leftrightarrow P) \to (Q \land (P \leftrightarrow Q))$

$F_5 : \neg R \leftrightarrow Q$

	P	Q	R	F_1	F_2	F_3	F_4	F_5		
1.	1	1	1	1	1	1	1	0		
2.	1	1	0	1	0	1	1	1		
3.	1	0	1	0	1	0	1	1		
4.	1	0	0	1	1	0	1	0		
5.	0	1	1	1	1	1	1	0		
6.	0	1	0	1	1	1	1	1		
7.	0	0	1	0	1	0	0	1		
8.	0	0	0	1	0	0	0	0		

(a) What is the Conjunctive Normal Form of $F_1 \to F_5$?

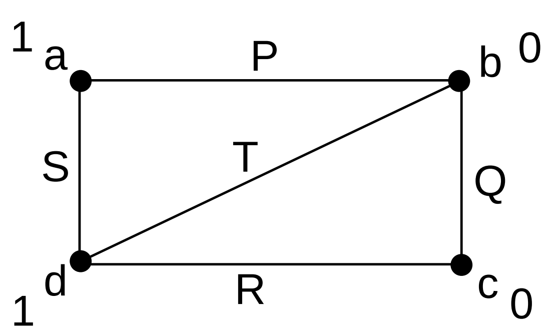
(b) Is the set $\{F_1, \neg F_2, F_3\}$ satisfiable? (Why?)

(c) Is the argument $\neg F_1,\ F_3 \therefore F_4$ valid? (Why?)

Problem 4

14 marks

Find the graph clauses for the labelled graph on the right, and state (with reasons) whether or not this set of clauses is satisfiable.



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Problem 5

10 marks

The following gives the first part of the completeness proof for the FŁ proof system. Fill in the reasons to justify Lemma D.0.5.

The Frege-Łukasiewicz Propositional Logic
Propositional Variables: $P, Q, \ldots$
Connectives: $\neg\,, \rightarrow$
Rule of inference: (modus ponens) $\dfrac{\mathsf{F},\ \mathsf{F} \rightarrow \mathsf{G}}{\mathsf{G}}$
Axiom schemata:
A1: $\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$
A2: $(\mathsf{F} \rightarrow (\mathsf{G} \rightarrow \mathsf{H})) \rightarrow ((\mathsf{F} \rightarrow \mathsf{G}) \rightarrow (\mathsf{F} \rightarrow \mathsf{H}))$
A3: $(\neg\,\mathsf{F} \rightarrow \neg\,\mathsf{G}) \rightarrow (\mathsf{G} \rightarrow \mathsf{F})$

Lemma A: If F is an axiom then $\vdash \mathsf{F}$.

Lemma B: If $\mathsf{F} \in \mathcal{S}$ then $\mathcal{S} \vdash \mathsf{F}$.

Lemma D.0.5: $\vdash \mathsf{F} \rightarrow \mathsf{F}$.

Proof. The following gives a derivation of $\mathsf{F} \rightarrow \mathsf{F}$:

1. $\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})$	____________________
2. $(\mathsf{F} \rightarrow ((\mathsf{F} \rightarrow \mathsf{F}) \rightarrow \mathsf{F})) \rightarrow ((\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F}))$	____________________
3. $(\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})) \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$	____________________
4. $\mathsf{F} \rightarrow (\mathsf{F} \rightarrow \mathsf{F})$	____________________
5. $\mathsf{F} \rightarrow \mathsf{F}$.	____________________

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Problem 6

16 marks

(a) Without assuming any set of connectives is adequate, show that $\mathcal{C} = \{\wedge,\ \neg\}$ is adequate.

(b) Use (a) to show that the Sheffer stroke $|$ is adequate.