The following output was generated by S.O.B.B:

14	$(x \cdot y) + (x \cdot z)$	$=$	$x \cdot (y + z)$	
100	$(x \cdot y) + (z \cdot x)$	$=$	$x \cdot (y + z)$	
127	$x \cdot (y \cdot (z + x))$	$=$	$y \cdot x$	
628	$x \cdot (y + (z \cdot (u + x)))$	$=$	$x \cdot (y + z)$	[para_from 127.1.1, 14.1.1.2; demod 100 ;flip.1]

Give the details of how S.O.B.B. found equation 628. (Follow the format in the supplementary page on Critical Pairs and Equational Theorem Proving.)

DETAILS: We want to find the critical pair equation for the following pair of equations by unifiying the boxed terms:

$$\begin{array}{lrcl} 127 & \boxed{x \cdot (y \cdot (z + x))} & = & y \cdot x \\ 14 & (x \cdot y) + \boxed{(x \cdot z)} & = & x \cdot (y + z) \end{array}$$

The first thing to do is to make the variables disjoint:

$$\begin{array}{lrcl} 127 & \boxed{x \cdot (y \cdot (z + x))} & = & y \cdot x \\ 14 & (X \cdot Y) + \boxed{(X \cdot Z)} & = & X \cdot (Y + Z) \end{array}$$

The most general unifier of the boxed terms is

$$\mu = \begin{pmatrix} X \leftarrow x \\ Z \leftarrow y \cdot (z + x) \end{pmatrix}$$

Applying this to 127 and 14 gives

$$\begin{array}{lrcl} 127 & \boxed{x \cdot (y \cdot (z + x))} & = & y \cdot x \\ 14 & (x \cdot Y) + \boxed{(x \cdot (y \cdot (z + x))} & = & x \cdot (Y + (y \cdot (z + x))) \end{array}$$

From this we have the Critical Pair Equation:

$$(**) \qquad (x \cdot Y) + (y \cdot x) \;=\; x \cdot (Y + (y \cdot (z + x)))$$

By applying 100 to the left side of (**) we obtain:

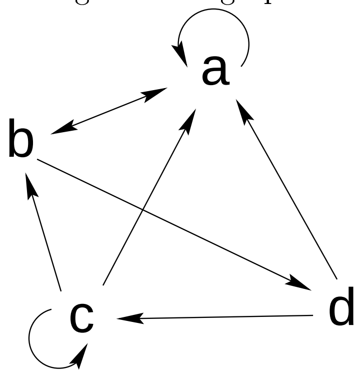
$$x \cdot (Y + y) \;=\; x \cdot (Y + (y \cdot (z + x)))$$

Flip, and change the variables, to get

$$x \cdot (y + (z \cdot (u + x))) \;=\; x \cdot (y + z)$$

Structures

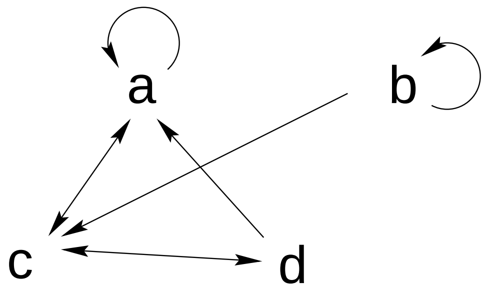
Fill in the table for the following directed graph:



	a	b	c	d
a	1	1	0	0
b	1	0	0	1
c	1	1	1	0
d	1	0	1	0

Draw the directed graph for the following table:

	a	b	c	d
a	1	0	1	0
b	0	1	1	0
c	1	0	0	1
d	1	0	1	0



Herbrand Universe

Suppose that a first-order languge has only a constant symbol a, a binary function symbol f, and a binary relation symbol r.

List the elements of the second level S_2 of the Herbrand universe:

$$S_2 = \underline{\{a,\ faa,\ ffaaa,\ fafaa,\ ffaafaa\}}$$

List all of the ground clauses that one can make from the symmetry clause $\{\neg rxy,\ ryx\}$ using just the portion of the Herbrand universe called S_1.

ANSWER:

$$\{\neg raa,\ raa\}$$
$$\{\neg rafaa,\ rfaaa\}$$
$$\{\neg rfaaa,\ rafaa\}$$
$$\{\neg rfaafaa,\ rfaafaa\}$$

Clauses

Let **S** be the structure on $\{0,1,2\}$ with a unary operation defined by $fx = x+1 \pmod 3$. The relation $<$ is the usual, that is, $0 < 1 < 2$. Given the literals

$$\mathsf{L}_1 \text{ is } \neg(fx < y) \qquad \mathsf{L}_2 \text{ is } x < fy$$

determine all pairs (a,b) that satisfy the clause $\mathsf{C} = \{\mathsf{L}_1, \mathsf{L}_2\}$ in **S** by filling in the following:

x	y	fx	fy	$fx < y$	$\neg(fx < y)$	$x < fy$	C
0	0	1	1	0	1	1	1
0	1	1	2	0	1	1	1
0	2	1	0	1	0	0	0
1	0	2	1	0	1	0	1
1	1	2	2	0	1	1	1
1	2	2	0	0	1	0	1
2	0	0	1	0	1	0	1
2	1	0	2	1	0	0	0
2	2	0	0	1	0	0	0

Does **S** satisfy the clause C? NO

Opp-unification: Suppose our first-order language has a unary relation symbol r and a binary operation symbol g. Show that the two clauses $\{rgxgyx,\ rggyzw\}$ and $\{\neg rgxggyzw\}$ are opp-unifiable, and find the most general opp-unifier (σ_1, σ_2). [Change x, y, z, w in the second clause to X, Y, Z, W.]

r	g	x	g	y	x											
r	g	g	y	z	w											
r	g	X	g	g	Y	Z	W									

$\downarrow \quad (x \leftarrow X)$

r	g	X	g	y	X											
r	g	g	y	z	w											
r	g	X	g	g	Y	Z	W									

$\downarrow \quad (X \leftarrow gyz)$

r	g	g	y	z	g	y	g	y	z							
r	g	g	y	z	w											
r	g	g	y	z	g	g	Y	Z	W							

$\downarrow \quad (w \leftarrow gygyz)$

r	g	g	y	z	g	y	g	y	z							
r	g	g	y	z	g	y	g	y	z							
r	g	g	y	z	g	g	Y	Z	W							

$\downarrow \quad (y \leftarrow gYZ)$

r	g	g	g	Y	Z	z	g	g	Y	Z	g	g	Y	Z	z	
r	g	g	g	Y	Z	z	g	g	Y	Z	g	g	Y	Z	z	
r	g	g	g	Y	Z	z	g	g	Y	Z	W					

$\downarrow (W \leftarrow ggYZz)$

r	g	g	g	Y	Z	z	g	g	Y	Z	g	g	Y	Z	z	
r	g	g	g	Y	Z	z	g	g	Y	Z	g	g	Y	Z	z	
r	g	g	g	Y	Z	z	g	g	Y	Z	g	g	Y	Z	z	

so $x \leftarrow ggYZz$, $w \leftarrow ggYZggYZz$, $X \leftarrow ggYZz$, $y \leftarrow gYZ$, $W \leftarrow ggYZz$.

Thus, with a, b, c being *any* three distinct variables,

$$\sigma_1 = \begin{pmatrix} x & \leftarrow & ggabc \\ y & \leftarrow & gab \\ z & \leftarrow & c \\ w & \leftarrow & ggabggabc \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} x & \leftarrow & ggabc \\ y & \leftarrow & a \\ z & \leftarrow & b \\ w & \leftarrow & ggabc \end{pmatrix}$$

Resolution Theorem Proving

For f a unary function symbol and r a binary relation symbol fill in the reasons for the following resolution derivation:

1.	$\{rxfx\}$	given
2.	$\{\neg rf00\}$	given
3.	$\{\neg rxy,\ ryx\}$	given
4.	$\{rfxx\}$	Res 1, 3
5.	$\{\ \}$	Res 2, 4

The following uses ground clauses from the given clauses 1–3 of the previous problem. Identify the source of each ground clause (e.g., clause 3) that is a ground instance of one of the above given clauses, and give reasons (i.e., line numbers) for the resolution steps:

1.	$\{r0f0\}$	Clause1
2.	$\{\neg r0f0,\ rf00\}$	Clause 3
3.	$\{\neg rf00\}$	Clause 2
4.	$\{\neg r0f0\}$	Res 2, 3
5.	$\{\ \}$	Res 1, 4