Unification

In each of the following you are given a pair of terms that you are to test for being unifiable, and if they are unifiable, give the most general unifier.

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(1) $(x \cdot (y+z)) + (x+y)$ and $((u+v)\cdot w) + u$

$+$	$\cdot$	x	$+$	y	z	$+$	x	y								
$+$	$\cdot$	$+$	u	v	w	u										

$\downarrow\ x \leftarrow u+v$

$+$	$\cdot$	$+$	u	v	$+$	y	z	$+$	$+$	u	v	y				
$+$	$\cdot$	$+$	u	v	w	u										

$\downarrow\ w \leftarrow y+z$

$+$	$\cdot$	$+$	u	v	$+$	y	z	$+$	$+$	u	v	y				
$+$	$\cdot$	$+$	u	v	$+$	y	z	u								

The original terms are not unifiable since the CSC fails at this step.

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(2) $(x \cdot (y+z)) + z$ and $((u+v)\cdot v) + 1$

$+$	$\cdot$	x	$+$	y	z	z										
$+$	$\cdot$	$+$	u	v	v	1										

$\downarrow\ x \leftarrow u+v$

$+$	$\cdot$	$+$	u	v	$+$	y	z	z								
$+$	$\cdot$	$+$	u	v	v	1										

$\downarrow\ v \leftarrow y+z$

$+$	$\cdot$	$+$	u	$+$	y	z	$+$	y	z	z						
$+$	$\cdot$	$+$	u	$+$	y	z	$+$	y	z	1						

$\downarrow\ z \leftarrow 1$

$+$	$\cdot$	$+$	u	$+$	y	1	$+$	y	1	1						
$+$	$\cdot$	$+$	u	$+$	y	1	$+$	y	1	1						

The terms are unifiable, and the most general unifier is $\mu = \begin{pmatrix} x \leftarrow u+(y+1) \\ v \leftarrow y+1 \\ z \leftarrow 1 \end{pmatrix}$

For the Normal Form TRS given by

$$\mathcal{R} \approx \{fffx \longrightarrow fx\}$$

find the normal forms for the following terms:

Term	Normal Form
fx	fx
ffy	ffy
$fffz$	fz
$ffffu$	ffu
$fffffv$	fv

For the Normal Form TRS given by

$$\mathcal{R} \approx \{gfx \longrightarrow fgx,\ ffx \longrightarrow gx,\ gggx \longrightarrow fx\}$$

find the normal forms for the following terms:

Term	Normal Form
$gfgx$	$fggx$
$gffgy$	fy
$gfgfgz$	fgz
$ffggu$	fu
$fgfgfv$	gv

For the Normal Form TRS given by

$$\mathcal{R} \approx \{(x+y)+z \longrightarrow x+z\}$$

find the normal forms for the following terms:

Term	Normal Form
$(x+x)+x$	$x+x$
$(x+u)+(y+v)$	$x+(y+v)$
$(x+(u+v))+(v+u)$	$x+(v+u)$
$((x+w)+(x+u))+y$	$x+y$
$(x+y)+((y+z)+(z+w))$	$x+(y+(z+w))$

Indicate why the TRS

$$\mathcal{R} \approx \{x \cdot y \longrightarrow z + x\}$$

is not terminating for the term $x \cdot y$ by filling in a few steps of

$$x \cdot y \longrightarrow_{\mathcal{R}} \underline{\ (x \cdot y) + x\ } \longrightarrow_{\mathcal{R}} \underline{\ ((x \cdot y) + x) + x\ } \longrightarrow_{\mathcal{R}} \underline{\ (((x \cdot y) + x) + x) + x\ }$$

Indicate why the TRS

$$\mathcal{R} \approx \{(x + y) + z \longrightarrow (z + x) + y\}$$

is not terminating for the term $(x + y) + z$ by filling in a few steps of

$$(x + y) + z \longrightarrow_{\mathcal{R}} \underline{\ (z + x) + y\ } \longrightarrow_{\mathcal{R}} \underline{\ (y + z) + x\ } \longrightarrow_{\mathcal{R}} \underline{\ (x + y) + z\ }$$

Indicate why the terminating TRS

$$\mathcal{R} \approx \{x + fy \longrightarrow y + x,\ fx + y \longrightarrow x\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$fx + fy$	x
$fx + fy$	$x + y$

Indicate why the terminating TRS

$$\mathcal{R} \approx \{(x + y) + z \longrightarrow y + (x + z)\}$$

is not a normal form TRS by giving two different terminal forms:

Term	Terminal Form
$((x + y) + z) + w$	$z + (y + (x + w))$
$((x + y) + z) + w$	$z + (x + (y + w))$

Given the pair of term rewrite rules (with disjoint variables)

$$\underline{fgfx} \longrightarrow gfgx \quad \text{and} \quad gf\underline{fu} \longrightarrow fgu$$

find the critical pair that results from unifying the underlined subterms:(Show Work)

Answer: $gfgfgx,\ fggfx$

DETAILS: The most general unifier of $fgfx$ and fu is $\mu = (u \leftarrow gfx)$.

Applying this to both of the above rules gives

$$\begin{aligned} fgfx &\longrightarrow gfgx \\ gffgfx &\longrightarrow fggfx \end{aligned}$$

To make the two left sides equal we do a replacement with the first rule:

$$\begin{aligned} gffgfx &\longrightarrow gfgfgx \\ gffgfx &\longrightarrow fggfx \end{aligned}$$

Thus the critical pair is $gfgfgx,\ fggfx$

Given the pair of term rewrite rules (with disjoint variables)

$$\underline{(y+z)}\cdot x \longrightarrow (y\cdot x)+(z\cdot x) \quad \text{and} \quad \underline{(u+v)+w} \longrightarrow u+(v+w)$$

find the critical pair that results from unifying the underlined subterms:(Show Work)

Answer: $((u+v)\cdot x)+(w\cdot x),\ (u+(v+w))\cdot x.$

DETAILS:

The most general unifier of $y+z$ and $(u+v)+w$ is $\mu = \begin{pmatrix} y \leftarrow u+v \\ z \leftarrow w \end{pmatrix}$.

Applying this to both of the above rules gives

$$\begin{array}{rcl} ((u+v)+w)\cdot x & \longrightarrow & ((u+v)\cdot x)+(w\cdot x) \\ (u+v)+w & \longrightarrow & u+(v+w) \end{array}$$

To make the two left sides equal we do a replacement with the second rule:

$$\begin{array}{rcl} ((u+v)+w)\cdot x & \longrightarrow & ((u+v)\cdot x)+(w\cdot x) \\ ((u+v)+w)\cdot x & \longrightarrow & (u+(v+w))\cdot x \end{array}$$

Thus the critical pair is $((u+v)\cdot x)+(w\cdot x),\ (u+(v+w))\cdot x$.