Substitution/Replacement

In each of the following inferences you are to choose the answer that gives the most ways in which the inference can be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1. $$\frac{x + x \;\approx\; 0}{(x+x)+(x+x) \;\approx\; 0}$$ Substitution

2. $$\frac{x + (y+z) \;\approx\; (x+y)+z}{(x+y)+z \;\approx\; x+(y+z)}$$ Neither

3. $$\frac{x \cdot y \;\approx\; x \cdot z}{(y\cdot z)\cdot(x\cdot y) \;\approx\; (y\cdot z)\cdot(x\cdot z)}$$ Both

4. $$\frac{x \cdot y \;\approx\; y \cdot x}{y\cdot(x\cdot y) \;\approx\; y\cdot(y\cdot x)}$$ Replacement

5. $$\frac{x + y \;\approx\; u + v}{y+(x+y) \;\approx\; y+(u+v)}$$ Both

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Find a two element **counterexample** to the following equational argument:

$$\frac{x \cdot y \;\approx\; u \cdot v}{x\cdot(y+z) \;\approx\; (x\cdot y)+(x\cdot z)}$$

$+$	a	b
a	b	$\star$
b	$\star$	$\star$

$\cdot$	a	b
a	a	a
b	a	a

Find a two element **counterexample** to the following argument:

$$\frac{fg(x) \;\approx\; gf(x)}{f(x) \;\approx\; g(x)}$$

	f
a	a
b	b

	g
a	b
b	a

Give a derivation using Birkhoff's Rules, stating reasons for your steps, to show that

$$x \cdot y \approx u \cdot v \vdash (x \cdot x) + (y \cdot y) \approx (y \cdot y) + (x \cdot x).$$

The following is a derivation:

1. $x \cdot y \approx u \cdot v$	Given
2. $x \cdot x \approx y \cdot y$	1 Subs
3. $(x \cdot x) + (y \cdot y) \approx (y \cdot y) + (y \cdot y)$	2 Repl
4. $y \cdot y \approx x \cdot x$	2 Symm
5. $(y \cdot y) + (y \cdot y) \approx (y \cdot y) + (x \cdot x)$	4 Repl
6. $(x \cdot x) + (y \cdot y) \approx (y \cdot y) + (x \cdot x)$	3,5 Trans

Fill in the reasons for the steps in the following derivation. [This is part of a derivation found by an automated theorem prover.]

1.	$x + 0$	$\approx$	x	given
2.	$x + (-x)$	$\approx$	0	given
3.	$x + y$	$\approx$	$y + x$	given
4.	$(x + y) + z$	$\approx$	$x + (y + z)$	given
5.	$x \cdot (y + z)$	$\approx$	$(x \cdot y) + (x \cdot z)$	given
6.	$(x + y) \cdot z$	$\approx$	$(x \cdot z) + (y \cdot z)$	given
7.	$x \cdot x$	$\approx$	x	given
8.	$(x \cdot y) + (x \cdot z)$	$\approx$	$x \cdot (y + z)$	5 Symm
9.	$(x \cdot z) + (y \cdot z)$	$\approx$	$(x + y) \cdot z$	6 Symm
10.	$(-x) + x$	$\approx$	$x + (-x)$	3 Subs
11.	$(-x) + x$	$\approx$	0	10, 2 Trans
12.	$(x + (-x)) + y$	$\approx$	$x + ((-x) + y)$	4 Subs
13.	$x + ((-x) + y)$	$\approx$	$(x + (-x)) + y$	12 Symm
14.	$(x + (-x)) + y$	$\approx$	$0 + y$	2 Repl
15.	$0 + y$	$\approx$	$y + 0$	3 Subs
16.	$y + 0$	$\approx$	y	1 Subs
17.	$0 + y$	$\approx$	y	15, 16 Trans
18.	$(x + (-x)) + y$	$\approx$	y	14, 17 Trans
19.	$x + ((-x) + y)$	$\approx$	y	13, 18 Trans
20.	$((-x) + x) + y$	$\approx$	$(x + (-x)) + y$	11 Repl
21.	$((-x) + x) + y$	$\approx$	y	20, 18 Trans
22.	$((-x) + x) + y$	$\approx$	$(-x) + (x + y)$	4 Subs
23.	$(-x) + (x + y)$	$\approx$	$((-x) + x) + y$	22 Symm
24.	$(-x) + (x + y)$	$\approx$	y	23, 21 Trans
25.	$y + x$	$\approx$	$x + y$	3 Subs
26.	$(-x) + (y + x)$	$\approx$	$(-x) + (x + y)$	25 Repl
27.	$(-x) + (y + x)$	$\approx$	y	26, 24 Trans
28.	$(x \cdot x) + (x \cdot y)$	$\approx$	$x \cdot (x + y)$	8 Subs
29.	$(x \cdot x) + (x \cdot y)$	$\approx$	$x + (x \cdot y)$	7 Repl