A Resolution Derivation

Given the collection of 8 clauses

1. $\{Q, S\}$	2. $\{R, S\}$	3. $\{\neg P, Q\}$	4. $\{P, R\}$
5. $\{P, \neg Q\}$	6. $\{\neg P, \neg R\}$	7. $\{\neg Q, \neg S\}$	8. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution derivation:

9.	$\{Q, \neg R\}$	$1, 8$	13.	$\{Q\}$	$9, 11$
10.	$\{\neg Q, R\}$	$2, 7$	14.	$\{\neg Q\}$	$10, 12$
11.	$\{Q, R\}$	$3, 4$	15.	$\{\ \}$	$13, 14$
12.	$\{\neg Q, \neg R\}$	$5, 6$			

Is it possible to find an assignment of truth values for the propositional variables P, Q, R, S that will satisfy the original eight clauses? (if yes, give one) $\boxed{\text{NO}}$

Given the collection $\mathcal{S}$ of 6 clauses

1. $\{P, \neg Q\}$	2. $\{Q, S\}$	3. $\{P, R\}$
4. $\{\neg P, \neg R\}$	5. $\{\neg Q, \neg S\}$	6. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution steps:

7.	$\{Q, \neg R\}$	$2, 6$	14.	$\{P, S\}$	$1, 2$
8.	$\{P, \neg S\}$	$3, 6$	15.	$\{P, \neg R\}$	$1, 7$
9.	$\{S, \neg S\}$	$2, 5$	16.	$\{P\}$	$3, 15$
10.	$\{Q, \neg Q\}$	$2, 5$	17.	$\{\neg R, S\}$	$2, 11$
11.	$\{\neg Q, \neg R\}$	$1, 4$	18.	$\{P, Q\}$	$2, 8$
12.	$\{P, \neg P\}$	$3, 4$	19.	$\{\neg R\}$	$4, 15$
13.	$\{R, \neg R\}$	$3, 4$			

Can you obtain any other clauses by resolution? $\boxed{\text{NO}}$

What does this say about the satisfiability of $\mathcal{S}$? $\boxed{\mathcal{S} \text{ is satisfiable.}}$

Apply the Davis-Putnam Procedure to the First Problem, showing just the ${\mathcal{S}_i}'$ and $\mathcal{U}_i$ steps (as done for the resolution on R below).

Resolution on R:

$${\mathcal{S}_1}'\colon\ \{Q,S\}\ \overset{(1)}{\{R,S\}}\ \{\neg P,Q\}\ \overset{(2)}{\{P,R\}}\ \{P,\neg Q\}\ \overset{(3)}{\{\neg P,\neg R\}}\ \{\neg Q,\neg S\}\ \overset{(4)}{\{\neg R,\neg S\}}$$

$$\mathcal{U}_1\colon\ \overset{(1,3)}{\{\neg P,S\}}\ \overset{(1,4)}{\{S,\neg S\}}\ \overset{(2,3)}{\{P,\neg P\}}\ \overset{(2,4)}{\{P,\neg S\}}$$

Resolution on P:

$${\mathcal{S}_2}'\colon\ \{Q,S\}\ \overset{(1)}{\{\neg P,Q\}}\ \overset{(2)}{\{P,\neg Q\}}\ \{\neg Q,\neg S\}\ \overset{(3)}{\{\neg P,S\}}\ \overset{(4)}{\{P,\neg S\}}$$

$$\mathcal{U}_2\colon\ \overset{(1,2)}{\{Q,\neg Q\}}\ \overset{(1,4)}{\{Q,\neg S\}}\ \overset{(2,3)}{\{\neg Q,S\}}\ \overset{(3,4)}{\{S,\neg S\}}$$

Resolution on Q:

$${\mathcal{S}_3}'\colon\ \overset{(1)}{\{Q,S\}}\ \overset{(2)}{\{\neg Q,\neg S\}}\ \overset{(3)}{\{Q,\neg S\}}\ \overset{(4)}{\{\neg Q,S\}}$$

$$\mathcal{U}_3\colon\ \overset{(1,2)}{\{S,\neg S\}}\ \overset{(1,4)}{\{S\}}\ \overset{(2,3)}{\{\neg S\}}$$

Resolution on S:

$${\mathcal{S}_4}'\colon\ \overset{(1)}{\{S\}}\ \overset{(2)}{\{\neg S\}}$$

$$\mathcal{U}_4\colon\ \overset{(1,2)}{\{\ \}}$$

Given the collection of five Horn clauses

1. $\{\neg P, Q\}$ 2. $\{P\}$ 3. $\{\neg R\}$ 4. $\{S, \neg Q\}$ 5. $\{\neg S, R\}$

find all clauses that can be derived using unit resolution:

	Clause	Reason
6.	$\{Q\}$	$1, 2$
7.	$\{\neg S\}$	$3, 5$
8.	$\{S\}$	$4, 6$
9.	$\{\neg Q\}$	$4, 7$
10.	$\{\neg P\}$	$1, 9$
11.	$\{R\}$	$5, 8$
12.	$\{\ \}$	$7, 8$

Given the clauses $\{\neg P, Q, R\}$ and $\{\neg Q, R, S\}$ prove, using just the definitions, that if $\vec{e}$ is a truth evaluation of P, Q, R, S that makes the two clauses true then it also makes the clause $\{\neg P, R, S\}$, obtained by resolving the two over Q, true.

First note that $\vec{e}$ makes Q true or it makes Q false.

Suppose $\vec{e}$ makes Q true. Then, as $\vec{e}$ makes the second clause $\{\neg Q, R, S\}$ true it follows that it must make $\{R, S\}$ true, and thus it makes the resolvent $\{\neg P, R, S\}$ true.

On the other hand, suppose that $\vec{e}$ makes Q false. Then, as $\vec{e}$ makes the first clause $\{\neg P, Q, R\}$ true it follows that it must make $\{\neg P, R\}$ true, and thus it makes the resolvent $\{\neg P, R, S\}$ true.

In either case, $\vec{e}$ makes the resolvent true.

Consider the propositional argument:

$$\begin{array}{ll} \mathsf{F}_1: & (P \to \neg Q) \to R \\ \mathsf{F}_2: & P \vee \neg(Q \to R) \\ \hline \mathsf{F}: & P \leftrightarrow (Q \leftrightarrow R) \end{array}$$

Give the conjunctive normal form for each of the following formulas:

F_1 : $(\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

F_2 : $(P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)$

$\neg\mathsf{F}$: $(\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$

From this derive a set $\mathcal{S}$ of clauses such that $\mathsf{F}_1, \mathsf{F}_2 \therefore \mathsf{F}$ is valid iff $\neg\,\mathsf{Sat}(\mathcal{S})$. $\mathcal{S}$ has the clauses:

1. $\neg P \vee Q \vee R$
2. $P \vee \neg Q \vee R$
3. $P \vee Q \vee R$
4. $P \vee \neg Q \vee \neg R$
5. $P \vee Q \vee \neg R$
6. $\neg P \vee \neg Q \vee \neg R$

Is $\mathcal{S}$ satisfiable? (Reasons)

$\boxed{\text{YES, by setting } PQR \text{ equal to 110, or to 101.}}$

Is the original argument valid? (Reasons)

$\boxed{\text{NO, as the set } \mathcal{S} \text{ of clauses (1–6) is satisfiable.}}$