## A Resolution Derivation

Given the collection of 8 clauses

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | $\{Q, S\}$ | 2. | $\{R, S\}$ | 3. | $\{\neg P, Q\}$ | 4. | $\{P, R\}$ |
| 5. | $\{P, \neg Q\}$ | 6. | $\{\neg P, \neg R\}$ | 7. | $\{\neg Q, \neg S\}$ | 8. | $\{\neg R, \neg S\}$ |

fill in the reasons for the following resolution derivation:

| | | | | | |
|---|---|---|---|---|---|
| 9. | $\{Q, \neg R\}$ | $1, 8$ | 13. | $\{Q\}$ | $9, 11$ |
| 10. | $\{\neg Q, R\}$ | $2, 7$ | 14. | $\{\neg Q\}$ | $10, 12$ |
| 11. | $\{Q, R\}$ | $3, 4$ | 15. | $\{\ \}$ | $13, 14$ |
| 12. | $\{\neg Q, \neg R\}$ | $5, 6$ | | | |

Is it possible to find an assignment of truth values for the propositional variables $P, Q, R, S$ that will satisfy the original eight clauses? (if yes, give one) $\boxed{\text{NO}}$

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Given the collection $\mathcal{S}$ of 6 clauses

| | | | | | |
|---|---|---|---|---|---|
| 1. | $\{P, \neg Q\}$ | 2. | $\{Q, S\}$ | 3. | $\{P, R\}$ |
| 4. | $\{\neg P, \neg R\}$ | 5. | $\{\neg Q, \neg S\}$ | 6. | $\{\neg R, \neg S\}$ |

fill in the reasons for the following resolution steps:

| | | | | | |
|---|---|---|---|---|---|
| 7. | $\{Q, \neg R\}$ | $2, 6$ | 14. | $\{P, S\}$ | $1, 2$ |
| 8. | $\{P, \neg S\}$ | $3, 6$ | 15. | $\{P, \neg R\}$ | $1, 7$ |
| 9. | $\{S, \neg S\}$ | $2, 5$ | 16. | $\{P\}$ | $3, 15$ |
| 10. | $\{Q, \neg Q\}$ | $2, 5$ | 17. | $\{\neg R, S\}$ | $2, 11$ |
| 11. | $\{\neg Q, \neg R\}$ | $1, 4$ | 18. | $\{P, Q\}$ | $2, 8$ |
| 12. | $\{P, \neg P\}$ | $3, 4$ | 19. | $\{\neg R\}$ | $4, 15$ |
| 13. | $\{R, \neg R\}$ | $3, 4$ | | | |

Can you obtain any other clauses by resolution? $\boxed{\text{NO}}$

What does this say about the satisfiability of $\mathcal{S}$? $\boxed{\mathcal{S}\text{ is satisfiable.}}$

Apply the Davis-Putnam Procedure to the First Problem, showing just the $\mathcal{S}_i{}'$ and $\mathcal{U}_i$ steps (as done for the resolution on $R$ below).

## Resolution on $R$:

$$\mathcal{S}_1{}': \ \{Q,S\} \ \overset{(1)}{\{R,S\}} \ \{\neg P,Q\} \ \overset{(2)}{\{P,R\}} \ \{P,\neg Q\} \ \overset{(3)}{\{\neg P,\neg R\}} \ \{\neg Q,\neg S\} \ \overset{(4)}{\{\neg R,\neg S\}}$$

$$\mathcal{U}_1: \ \overset{(1,3)}{\{\neg P,S\}} \ \overset{(1,4)}{\{S,\neg S\}} \ \overset{(2,3)}{\{P,\neg P\}} \ \overset{(2,4)}{\{P,\neg S\}}$$

## Resolution on $P$:

$$\mathcal{S}_2{}': \ \{Q,S\} \ \overset{(1)}{\{\neg P,Q\}} \ \overset{(2)}{\{P,\neg Q\}} \ \{\neg Q,\neg S\} \ \overset{(3)}{\{\neg P,S\}} \ \overset{(4)}{\{P,\neg S\}}$$

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$$\mathcal{U}_2: \ \overset{(1,2)}{\{Q,\neg Q\}} \ \overset{(1,4)}{\{Q,\neg S\}} \ \overset{(2,3)}{\{\neg Q,S\}} \ \overset{(3,4)}{\{S,\neg S\}}$$

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## Resolution on $Q$:

$$\mathcal{S}_3{}': \ \overset{(1)}{\{Q,S\}} \ \overset{(2)}{\{\neg Q,\neg S\}} \ \overset{(3)}{\{Q,\neg S\}} \ \overset{(4)}{\{\neg Q,S\}}$$

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$$\mathcal{U}_3: \ \overset{(1,2)}{\{S,\neg S\}} \ \overset{(1,4)}{\{S\}} \ \overset{(2,3)}{\{\neg S\}}$$

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## Resolution on $S$:

$$\mathcal{S}_4{}': \ \overset{(1)}{\{S\}} \ \overset{(2)}{\{\neg S\}}$$

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$$\mathcal{U}_4: \ \overset{(1,2)}{\{\ \}}$$

Given the collection of five Horn clauses

$$1.\ \{\neg P, Q\} \quad 2.\ \{P\} \quad 3.\ \{\neg R\} \quad 4.\ \{S, \neg Q\} \quad 5.\ \{\neg S, R\}$$

find all clauses that can be derived using unit resolution:

| | Clause | Reason |
|---|---|---|
| 6. | $\{Q\}$ | $1, 2$ |
| 7. | $\{\neg S\}$ | $3, 5$ |
| 8. | $\{S\}$ | $4, 6$ |
| 9. | $\{\neg Q\}$ | $4, 7$ |
| 10. | $\{\neg P\}$ | $1, 9$ |
| 11. | $\{R\}$ | $5, 8$ |
| 12. | $\{\ \}$ | $7, 8$ |

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Given the clauses $\{\neg P, Q, R\}$ and $\{\neg Q, R, S\}$ prove, using just the definitions, that if $\vec{e}$ is a truth evaluation of $P, Q, R, S$ that makes the two clauses true then it also makes the clause $\{\neg P, R, S\}$, obtained by resolving the two over $Q$, true.

First note that $\vec{e}$ makes $Q$ true or it makes $Q$ false.

Suppose $\vec{e}$ makes $Q$ true. Then, as $\vec{e}$ makes the second clause $\{\neg Q, R, S\}$ true it follows that it must make $\{R, S\}$ true, and thus it makes the resolvent $\{\neg P, R, S\}$ true.

On the other hand, suppose that $\vec{e}$ makes $Q$ false. Then, as $\vec{e}$ makes the first clause $\{\neg P, Q, R\}$ true it follows that it must make $\{\neg P, R\}$ true, and thus it makes the resolvent $\{\neg P, R, S\}$ true.

In either case, $\vec{e}$ makes the resolvent true.

Consider the propositional argument:

$$\begin{array}{ll} \mathsf{F}_1: & (P \to \neg Q) \to R \\ \mathsf{F}_2: & P \vee \neg(Q \to R) \\ \hline \mathsf{F}: & P \leftrightarrow (Q \leftrightarrow R) \end{array}$$

Give the conjunctive normal form for each of the following formulas:

$\mathsf{F}_1$ : $(\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

$\mathsf{F}_2$ : $(P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)$

$\neg\mathsf{F}$ : $(\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$

From this derive a set $\mathcal{S}$ of clauses such that $\mathsf{F}_1, \mathsf{F}_2 \therefore \mathsf{F}$ is valid iff $\neg\mathsf{Sat}(\mathcal{S})$. $\mathcal{S}$ has the clauses:

1. $\neg P \vee Q \vee R$
2. $P \vee \neg Q \vee R$
3. $P \vee Q \vee R$
4. $P \vee \neg Q \vee \neg R$
5. $P \vee Q \vee \neg R$
6. $\neg P \vee \neg Q \vee \neg R$

Is $\mathcal{S}$ satisfiable? (Reasons)

YES, by setting $PQR$ equal to 110, or to 101.

Is the original argument valid? (Reasons)

NO, as the set $\mathcal{S}$ of clauses (1–6) is satisfiable.