

Put the following formula in prenex form. Use capital letters when renaming variables. Show details of your work:

$$\forall u (u < y) \rightarrow \exists y \forall z [\neg(u + y < w + z) \rightarrow \neg \exists w (w \cdot y < w \cdot z)]$$


---

**Answer:**  $\exists U \exists Y \forall z \forall W ((U < y) \rightarrow [\neg(u + Y < w + z) \rightarrow \neg \exists W (W \cdot Y < W \cdot z)])$

---

DETAILS: First rename bound occurrences of  $u, y, z$ :

$$\forall U (U < y) \rightarrow \exists Y \forall z [\neg(u + Y < w + z) \rightarrow \neg \exists W (W \cdot Y < W \cdot z)]$$

Then proceed to pull the quantifiers to the front:

$$\exists U ((U < y) \rightarrow \exists Y \forall z [\neg(u + Y < w + z) \rightarrow \neg \exists W (W \cdot Y < W \cdot z)])$$

$$\exists U \exists Y \forall z ((U < y) \rightarrow [\neg(u + Y < w + z) \rightarrow \neg \exists W (W \cdot Y < W \cdot z)])$$

$$\exists U \exists Y \forall z ((U < y) \rightarrow [\neg(u + Y < w + z) \rightarrow \forall W \neg(W \cdot Y < W \cdot z)])$$

$$\exists U \exists Y \forall z \forall W ((U < y) \rightarrow [\neg(u + Y < w + z) \rightarrow \neg(W \cdot Y < W \cdot z)])$$

Convert the following argument into a set  $\mathcal{C}$  of clauses such that the argument is valid iff the set  $\mathcal{C}$  is not satisfiable. Use capital letters when changing the names of variables. Show details of your work.

$$\forall x \left( \exists y (x \cdot y < x) \rightarrow \exists z ((y \cdot z < x) \wedge \neg(y \cdot x < z)) \right) \quad \therefore \exists x \forall y (x \cdot y < y)$$


---

**Answer:**  $\{\neg(x \cdot Y < x), (y \cdot f x Y < x)\}$   
 $\{\neg(x \cdot Y < x), \neg(y \cdot x < f x Y)\}$  gives the clauses of  $\mathcal{C}$ .  
 $\{\neg(x \cdot g x < g x)\}$

---

DETAILS: There is one premiss and the conclusion. Changing bound variables in the premiss gives:

$$\forall x \left( \exists Y (x \cdot Y < x) \rightarrow \exists z ((y \cdot z < x) \wedge \neg(y \cdot x < z)) \right)$$

Then the prenex form is:

$$\forall x \forall Y \exists z \left( (x \cdot Y < x) \rightarrow ((y \cdot z < x) \wedge \neg(y \cdot x < z)) \right)$$

Skolemizing the prenex form of the premiss gives

$$\forall x \forall Y \left( (x \cdot Y < x) \rightarrow ((y \cdot f x Y < x) \wedge \neg(y \cdot x < f x Y)) \right)$$

Putting the matrix in conjunctive form gives

$$((\neg(x \cdot Y < x) \vee (y \cdot f x Y < x)) \wedge (\neg(x \cdot Y < x) \vee \neg(y \cdot x < f x Y))) \quad (1)$$

The negated conclusion is

$$\neg \exists x \forall y (x \cdot y < y)$$

which is equivalent to

$$\forall x \exists y \neg (x \cdot y < y)$$

Skolemizing the prenex form of the negated conclusion gives

$$\forall x \neg (x \cdot g x < g x)$$

The matrix in conjunctive form is given by

$$\neg (x \cdot g x < g x) \quad (2)$$

From (1) and (2) we have the three clauses given in the Answer above.