

Complete the following from Appendix B.

**Lemma B.0.18**  $(a \cdot b)^n = a^n \cdot b^n$ .

PROOF. By induction on  $n$ .

For  $n = 1$ :

$$\begin{aligned} (a \cdot b)^1 &= a \cdot b && \text{by } \boxed{\text{B.0.15 i}} \\ &= a^1 \cdot b^1. && \text{by } \boxed{\text{B.0.15 i}} \end{aligned}$$

Induction Hypothesis:  $(a \cdot b)^n = a^n \cdot b^n$ .

$$\begin{aligned} \text{Thus } (a \cdot b)^{n'} &= (a \cdot b)^n \cdot (a \cdot b) && \text{by } \boxed{\text{B.0.15 ii}} \\ &= ((a \cdot b)^n \cdot a) \cdot b && \text{by } \boxed{\text{B.0.14}} \\ &= (a \cdot (a \cdot b)^n) \cdot b && \text{by } \boxed{\text{B.0.11}} \\ &= (a \cdot (a^n \cdot b^n)) \cdot b && \text{by } \boxed{\text{Ind Hyp}} \\ &= ((a \cdot a^n) \cdot b^n) \cdot b && \text{by } \boxed{\text{B.0.14}} \\ &= ((a^n \cdot a) \cdot b^n) \cdot b && \text{by } \boxed{\text{B.0.11}} \\ &= (a^{n'} \cdot b^n) \cdot b && \text{by } \boxed{\text{B.0.15 ii}} \\ &= a^{n'} \cdot (b^n \cdot b) && \text{by } \boxed{\text{B.0.14}} \\ &= a^{n'} \cdot b^{n'}. && \text{by } \boxed{\text{B.0.15 ii}} \end{aligned}$$

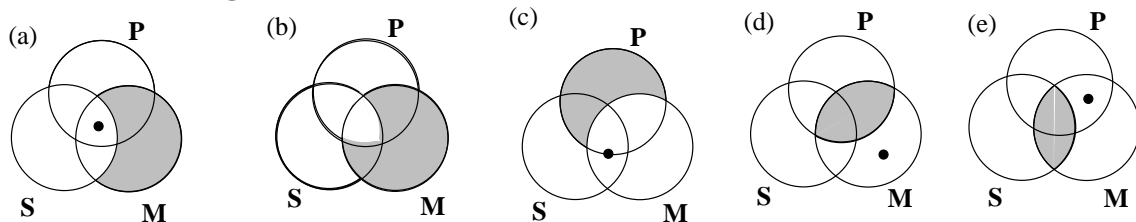
### Six Syllogisms

- |   |  |   |
|---|--|---|
| (1) $\frac{\text{All P is M.} \quad \text{Some M is S.}}{\text{Some S is P.}}$    | (2) $\frac{\text{All M is S.} \quad \text{Some P is M.}}{\text{Some S is P.}}$ | (3) $\frac{\text{Some M is not S.} \quad \text{No M is P.}}{\text{Some S is not P.}}$ |
| (4) $\frac{\text{Some P is M.} \quad \text{No S is M.}}{\text{Some S is not P.}}$ | (5) $\frac{\text{Some S is M.} \quad \text{All P is M.}}{\text{Some S is P.}}$ | (6) $\frac{\text{All M is P.} \quad \text{All M is S.}}{\text{Some P is S.}}$         |

For each of the above syllogisms find the **figure** and the **mood**, and determine if it is valid under Aristotle's Convention (AC), and if it is valid under the Modern Convention (MC).

	Figure	Mood	Valid under (AC)	Valid under (MC)
1	4	AII		
2	4	IAI	✓	✓
3	3	EOO		
4	2	IEO		
5	2	AII		
6	3	AAI	✓	

### Five Venn Diagrams



For each of the six syllogisms at the top find the Venn diagram for its premisses from the above and enter the answers in the empty boxes below:

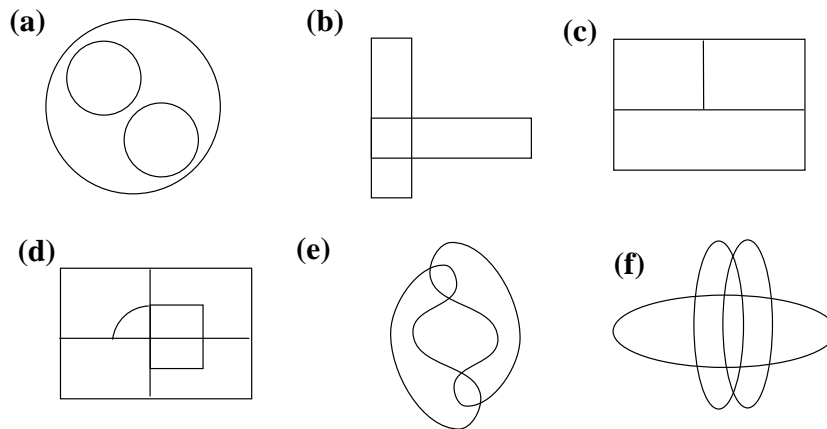
Syllogism	1	2	3	4	5	6
Venn Diagram	c	a	d	e	c	b

Translate each of the following four syllogisms into equational arguments.

$  \begin{array}{lcl}  (1) \text{ No P is M.} & PM = 0 & \\  \text{No M is S.} & MS = 0 & \\  \hline  \text{All S is P.} & SP' = 0 &   \end{array}  $	$  \begin{array}{lcl}  (2) \text{ No M is S.} & MS = 0 & \\  \text{All M is P.} & MP' = 0 & \\  \hline  \text{No S is P.} & SP = 0 &   \end{array}  $
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$  \begin{array}{lcl}  (3) \text{ No M is P.} & MP = 0 & \\  \text{All S is M.} & SM' = 0 & \\  \hline  \text{No S is P.} & SP = 0 &   \end{array}  $	$  \begin{array}{lcl}  (4) \text{ All P is M.} & PM' = 0 & \\  \text{No M is P.} & MP = 0 & \\  \hline  \text{All S is P.} & SP' = 0 &   \end{array}  $
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Which of the six diagrams below could qualify as **Venn diagrams**?



Answer:           c,d          

