Name: ID:

PMath 330

Assignment 9

Mark

Expressing facts about N in first-order language

Give formulas or sentences to express the following:

1. The maximum of x and y is z.

Answer:

2. x is of the form $2^m 7^n$

Answer:

3. There are an infinite number of primes of the form $n^2 + 1$.

Answer:

4. There are only finitely many primes of the form $n^2 + n + 1$.

Answer:

Let A be the sentence $\forall x\,(0 < x+1)$.

Let B be the sentence $\forall x \forall y \forall z\,\big((x<y) \wedge (y<z) \rightarrow (x+1<z)\big)$.

Let C be the sentence $\exists x\,(x \cdot x \approx 2)$.

Put a checkmark in each box for which the structure above satisfies the sentence to the left:

	N	**Z**	**Q**	**R**
A				
B				
C				

Give a propositional combination of the sentences A, B, C that is true only of **N** and **R** (be sure to simplify your answer, here and in the following questions):

Answer: ______________________

Give a propositional combination (using $\wedge, \vee, \rightarrow, \leftrightarrow, \neg$) of the sentences A, B, C that is true only of **Q**:

Answer: ______________________

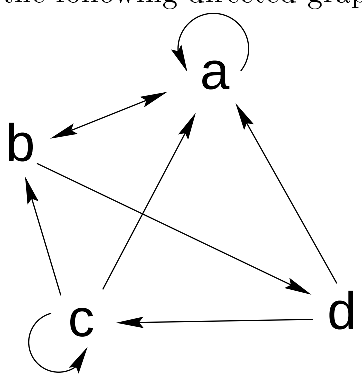
Give a propositional combination of the sentences A, B, C that is true only of **Z** and **Q**:

Answer: ______________________

Give a propositional combination of the sentences A, B, C that is true only of **Z** and **R**:

Answer: ______________________

Determine the binary relation defined by the formula $F(x,y) = \exists u\,(rxu \wedge ruy)$ on the following directed graph:



	a	b	c	d
a				
b				
c				
d				

Consider the following 5 sentences in the language of directed graphs:

F_1: $\forall x\,(rxx)$

F_2: $\exists x \forall y\,(rxy)$

F_3: $\forall x \forall y \forall z\,(rxy \wedge ryz \to rxz)$

F_4: $\forall x \forall y\big(rxy \to \exists z\,(rxz \wedge rzy)\big)$

F_5: $\forall x \exists y \forall z\,(rxy \wedge (rxz \to ryz))$

and the following two directed graphs:

$\mathbf{G}_1 = (G, r)$ where $G = \{0,1,2\}$ and $r = \{(0,1),\,(0,2),\,(1,2)\}$

$\mathbf{G}_2 = (N, \nmid)$, the nonnegative integers with the usual 'does not divide' relation.

Find the truth values of each of the above sentences in each of the above structures, and enter these values (0 or 1) in the table below.

[This question will be marked as follows: each correct answer is worth 1 mark; each incorrect answer receives a penalty of -1; each blank receives 0 marks. However, the lowest possible total mark for this question is 0.]

	F_1	F_2	F_3	F_4	F_5
$\mathbf{G}_1$					
$\mathbf{G}_2$					