

Name:

ID:

PMath 330

Assignment 9

Mark _____

Expressing facts about \mathbf{N} in first-order language

Give formulas or sentences to express the following:

1. The maximum of x and y is z .

Answer:

2. x is of the form $2^m 7^n$

Answer:

3. There are an infinite number of primes of the form $n^2 + 1$.

Answer:

4. There are only finitely many primes of the form $n^2 + n + 1$.

Answer:

Let A be the sentence $\forall x (0 < x + 1)$.

Let B be the sentence $\forall x \forall y \forall z ((x < y) \wedge (y < z) \rightarrow (x + 1 < z))$.

Let C be the sentence $\exists x (x \cdot x \approx 2)$.

Put a checkmark in each box for which the structure above satisfies the sentence to the left:

	N	Z	Q	R
A				
B				
C				

Give a propositional combination of the sentences A, B, C that is true only of **N** and **R** (be sure to simplify your answer, here and in the following questions):

Answer: _____

Give a propositional combination (using $\wedge, \vee, \rightarrow, \leftrightarrow, \neg$) of the sentences A, B, C that is true only of **Q**:

Answer: _____

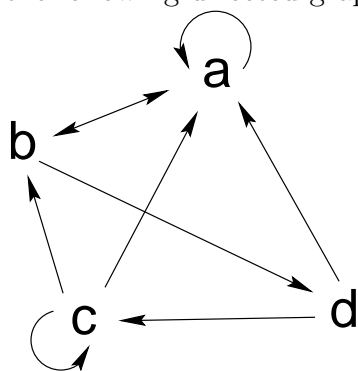
Give a propositional combination of the sentences A, B, C that is true only of **Z** and **Q**:

Answer: _____

Give a propositional combination of the sentences A, B, C that is true only of **Z** and **R**:

Answer: _____

Determine the binary relation defined by the formula $F(x, y) = \exists u (rxu \wedge ruy)$ on the following directed graph:



	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>a</i>				
<i>b</i>				
<i>c</i>				
<i>d</i>				

Consider the following 5 sentences in the language of directed graphs:

$$F_1: \forall x (rxx)$$

$$F_2: \exists x \forall y (rxy)$$

$$F_3: \forall x \forall y \forall z (rxy \wedge ryz \rightarrow rxz)$$

$$F_4: \forall x \forall y (rxy \rightarrow \exists z (rxz \wedge rzy))$$

$$F_5: \forall x \exists y \forall z (rxy \wedge (rxz \rightarrow ryz))$$

and the following two directed graphs:

$$G_1 = (G, r) \text{ where } G = \{0, 1, 2\} \text{ and } r = \{(0, 1), (0, 2), (1, 2)\}$$

$$G_2 = (N, \nmid), \text{ the nonnegative integers with the usual 'does not divide' relation.}$$

Find the truth values of each of the above sentences in each of the above structures, and enter these values (0 or 1) in the table below.

[This question will be marked as follows: each correct answer is worth 1 mark; each incorrect answer receives a penalty of -1; each blank receives 0 marks. However, the lowest possible total mark for this question is 0.]

	F_1	F_2	F_3	F_4	F_5
G_1					
G_2					