Name: ID:

PMath 330

Assignment 8

Mark ______

The following output was generated by S.O.B.B:

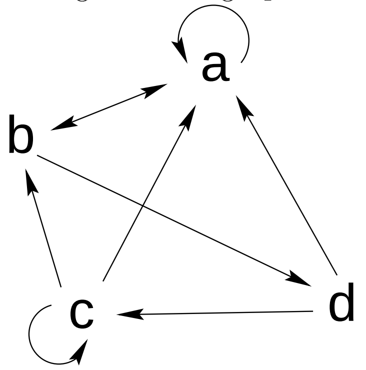
14	$(x \cdot y) + (x \cdot z)$	$=$	$x \cdot (y + z)$	
100	$(x \cdot y) + (z \cdot x)$	$=$	$x \cdot (y + z)$	
127	$x \cdot (y \cdot (z + x))$	$=$	$y \cdot x$	
628	$x \cdot (y + (z \cdot (u + x)))$	$=$	$x \cdot (y + z)$	[para_from 127.1.1, 14.1.1.2; demod 100 ;flip.1]

Give the details of how S.O.B.B. found equation 628. (Follow the format in the supplementary page on Critical Pairs and Equational Theorem Proving.)

DETAILS:

Structures

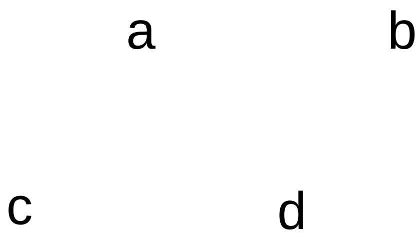
Fill in the table for the following directed graph:



	a	b	c	d
a				
b				
c				
d				

Draw the directed graph for the following table:

	a	b	c	d
a	1	0	1	0
b	0	1	1	0
c	1	0	0	1
d	1	0	1	0



Herbrand Universe

Suppose that a first-order languge has only a constant symbol a, a binary function symbol f, and a binary relation symbol r.

List the elements of the second level S_2 of the Herbrand universe:

S_2 = ______________________________

List all of the ground clauses that one can make from the symmetry clause $\{\neg rxy,\ ryx\}$ using just the portion of the Herbrand universe called S_1.

ANSWER:

Clauses

Let $\mathbf{S}$ be the structure on $\{0, 1, 2\}$ with a unary operation defined by $fx = x + 1 \pmod 3$. The relation $<$ is the usual, that is, $0 < 1 < 2$. Given the literals

$$\mathsf{L}_1 = \neg(fx < y) \qquad \mathsf{L}_2 = x < fy$$

determine all pairs (a, b) that satisfy the clause $\mathsf{C} = \{\mathsf{L}_1, \mathsf{L}_2\}$ in $\mathbf{S}$ by filling in the following:

x	y	fx	fy	$fx < y$	$\neg(fx < y)$	$x < fy$	C
0	0						
0	1						
0	2						
1	0						
1	1						
1	2						
2	0						
2	1						
2	2						

Does $\mathbf{S}$ satisfy the clause C? ________

Opp-unification: Suppose our first-order language has a unary relation symbol r and a binary operation symbol g. Show that the two clauses $\{rgxgyx,\ rggyzw\}$ and $\{\neg\, rgxggyzw\}$ are opp-unifiable, and find the most general opp-unifier (σ_1, σ_2). [Change x, y, z, w in the second clause to X, Y, Z, W.]

$$\text{Thus } \sigma_1 = \begin{pmatrix} x \leftarrow & \qquad\qquad \\ y \leftarrow & \\ z \leftarrow & \\ w \leftarrow & \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} x \leftarrow & \qquad\qquad \\ y \leftarrow & \\ z \leftarrow & \\ w \leftarrow & \end{pmatrix}$$

Resolution Theorem Proving

For f a unary function symbol and r a binary relation symbol fill in the reasons for the following resolution derivation:

1. $\{rxfx\}$	given
2. $\{\neg\, rf00\}$	given
3. $\{\neg\, rxy,\ ryx\}$	given
4. $\{rfxx\}$	____________
5. $\{\ \}$	____________

The following uses ground clauses from the given clauses 1–3 of the previous problem. Identify the source of each ground clause (e.g., clause 3) that is a ground instance of one of the above given clauses, and give reasons (i.e., line numbers) for the resolution steps:

1. $\{r0f0\}$	____________
2. $\{\neg\, r0f0,\ rf00\}$	____________
3. $\{\neg\, rf00\}$	____________
4. $\{\neg\, r0f0\}$	____________
5. $\{\ \}$	____________