Name: ID:

## PMath 330 Assignment 7

Mark

## Unification

In each of the following you are given a pair of terms that you are to test for being unifiable, and if they are unifiable, give the most general unifier.
(1) $(x \cdot(y+z))+(x+y)$ and $((u+v) \cdot w)+u$

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(2) $(x \cdot(y+z))+z$ and $((u+v) \cdot v)+1$


For the Normal Form TRS given by

$$
\mathcal{R} \approx\{f f f x \longrightarrow f x\}
$$

find the normal forms for the following terms:

| Term | Normal Form |
| :--- | :--- |
| $f x$ |  |
| $f f y$ |  |
| $f f f z$ | - |
| $f f f f u$ | - |
| $f f f f f v$ |  |

For the Normal Form TRS given by

$$
\mathcal{R} \approx\{g f x \longrightarrow f g x, f f x \longrightarrow g x, g g g x \longrightarrow f x\}
$$

find the normal forms for the following terms:

| Term | Normal Form |
| :--- | :--- |
| $g f g x$ |  |
| $g f f g y$ |  |
| gfgfgz |  |
| ffggu |  |
| fgfgfv |  |

For the Normal Form TRS given by

$$
\mathcal{R} \approx\{(x+y)+z \longrightarrow x+z\}
$$

find the normal forms for the following terms:

| Term | Normal Form |
| :--- | :--- |
| $(x+x)+x$ |  |
| $(x+u)+(y+v)$ |  |
| $(x+(u+v))+(v+u)$ |  |
| $((x+w)+(x+u))+y$ |  |
| $(x+y)+((y+z)+(z+w))$ |  |

Indicate why the TRS

$$
\mathcal{R} \approx\{x \cdot y \longrightarrow z+x\}
$$

is not terminating for the term $x \cdot y$ by filling in a few steps of
$x \cdot y \longrightarrow \mathcal{R}$ $\qquad$ $\longrightarrow \mathcal{R}$ $\qquad$ $\longrightarrow \mathcal{R}$

Indicate why the TRS

$$
\mathcal{R} \approx\{(x+y)+z \longrightarrow(z+x)+y\}
$$

is not terminating for the term $(x+y)+z$ by filling in a few steps of $(x+y)+z \longrightarrow_{\mathcal{R}} \longrightarrow \mathcal{R} \longrightarrow \mathcal{R}$

Indicate why the terminating TRS

$$
\mathcal{R} \approx\{x+f y \longrightarrow y+x, f x+y \longrightarrow x\}
$$

is not a normal form TRS by giving two different terminal forms:

| Term | Terminal Form |
| :--- | :--- |
| $f x+f y$ |  |
| $f x+f y$ |  |

Indicate why the terminating TRS

$$
\mathcal{R} \approx\{(x+y)+z \longrightarrow y+(x+z)\}
$$

is not a normal form TRS by giving two different terminal forms:

| Term | Terminal Form |
| :--- | :--- |
| $((x+y)+z)+w$ |  |
| $((x+y)+z)+w$ |  |

Given the pair of term rewrite rules (with disjoint variables)

$$
\underline{f g f x} \longrightarrow g f g x \quad \text { and } \quad g f \underline{f u} \longrightarrow f g u
$$

find the critical pair that results from unifying the underlined subterms:(Show Work)

Answer:

DETAILS:

Given the pair of term rewrite rules (with disjoint variables)

$$
\underline{(y+z)} \cdot x \longrightarrow(y \cdot x)+(z \cdot x) \quad \text { and } \quad \underline{(u+v)+w} \longrightarrow u+(v+w)
$$

find the critical pair that results from unifying the underlined subterms:(Show Work)

Answer:

DETAILS:

