Name: ID:

PMath 330

Assignment 6

Mark______

Substitution/Replacement

In each of the following inferences you are to choose the answer that gives the most ways in which the inference can be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1. $$\frac{x+x \;\approx\; 0}{(x+x)+(x+x) \;\approx\; 0}$$ ____________

2. $$\frac{x+(y+z) \;\approx\; (x+y)+z}{(x+y)+z \;\approx\; x+(y+z)}$$ ____________

3. $$\frac{x\cdot y \;\approx\; x\cdot z}{(y\cdot z)\cdot(x\cdot y) \;\approx\; (y\cdot z)\cdot(x\cdot z)}$$ ____________

4. $$\frac{x\cdot y \;\approx\; y\cdot x}{y\cdot(x\cdot y) \;\approx\; y\cdot(y\cdot x)}$$ ____________

5. $$\frac{x+y \;\approx\; u+v}{y+(x+y) \;\approx\; y+(u+v)}$$ ____________

Find a two element **counterexample** to the following equational argument:

$$\frac{x\cdot y \;\approx\; u\cdot v}{x\cdot(y+z) \;\approx\; (x\cdot y)+(x\cdot z)}$$

$+$	a	b
a		
b		

$\cdot$	a	b
a		
b		

Find a two element **counterexample** to the following argument:

$$\frac{fg(x) \;\approx\; gf(x)}{f(x) \;\approx\; g(x)}$$

	f
a	
b	

	g
a	
b	

Give a derivation using Birkhoff's Rules, stating reasons for your steps, to show that

$$x \cdot y \approx u \cdot v \vdash (x \cdot x) + (y \cdot y) \approx (y \cdot y) + (x \cdot x).$$

The following is a derivation:

1. $x \cdot y \approx u \cdot v$ Given

Fill in the reasons for the steps in the following derivation. [This is part of a derivation found by an automated theorem prover.]

1.	$x + 0$	$\approx$	x	given
2.	$x + (-x)$	$\approx$	0	given
3.	$x + y$	$\approx$	$y + x$	given
4.	$(x + y) + z$	$\approx$	$x + (y + z)$	given
5.	$x \cdot (y + z)$	$\approx$	$(x \cdot y) + (x \cdot z)$	given
6.	$(x + y) \cdot z$	$\approx$	$(x \cdot z) + (y \cdot z)$	given
7.	$x \cdot x$	$\approx$	x	given
8.	$(x \cdot y) + (x \cdot z)$	$\approx$	$x \cdot (y + z)$	
9.	$(x \cdot z) + (y \cdot z)$	$\approx$	$(x + y) \cdot z$	
10.	$(-x) + x$	$\approx$	$x + (-x)$	3 Subs
11.	$(-x) + x$	$\approx$	0	10, 2 Trans
12.	$(x + (-x)) + y$	$\approx$	$x + ((-x) + y)$	
13.	$x + ((-x) + y)$	$\approx$	$(x + (-x)) + y$	
14.	$(x + (-x)) + y$	$\approx$	$0 + y$	
15.	$0 + y$	$\approx$	$y + 0$	
16.	$y + 0$	$\approx$	y	
17.	$0 + y$	$\approx$	y	
18.	$(x + (-x)) + y$	$\approx$	y	
19.	$x + ((-x) + y)$	$\approx$	y	
20.	$((-x) + x) + y$	$\approx$	$(x + (-x)) + y$	
21.	$((-x) + x) + y$	$\approx$	y	
22.	$((-x) + x) + y$	$\approx$	$(-x) + (x + y)$	
23.	$(-x) + (x + y)$	$\approx$	$((-x) + x) + y$	
24.	$(-x) + (x + y)$	$\approx$	y	
25.	$y + x$	$\approx$	$x + y$	
26.	$(-x) + (y + x)$	$\approx$	$(-x) + (x + y)$	
27.	$(-x) + (y + x)$	$\approx$	y	
28.	$(x \cdot x) + (x \cdot y)$	$\approx$	$x \cdot (x + y)$	
29.	$(x \cdot x) + (x \cdot y)$	$\approx$	$x + (x \cdot y)$	