

Name:

ID:

PMath 330

Assignment 6

Mark _____

Substitution/Replacement

In each of the following inferences you are to choose the answer that gives the most ways in which the inference can be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1.
$$\frac{x + x \approx 0}{(x + x) + (x + x) \approx 0} \quad \underline{\hspace{2cm}}$$
2.
$$\frac{x + (y + z) \approx (x + y) + z}{(x + y) + z \approx x + (y + z)} \quad \underline{\hspace{2cm}}$$
3.
$$\frac{x \cdot y \approx x \cdot z}{(y \cdot z) \cdot (x \cdot y) \approx (y \cdot z) \cdot (x \cdot z)} \quad \underline{\hspace{2cm}}$$
4.
$$\frac{x \cdot y \approx y \cdot x}{y \cdot (x \cdot y) \approx y \cdot (y \cdot x)} \quad \underline{\hspace{2cm}}$$
5.
$$\frac{x + y \approx u + v}{y + (x + y) \approx y + (u + v)} \quad \underline{\hspace{2cm}}$$

Find a two element **counterexample** to the following equational argument:

$\frac{x \cdot y \approx u \cdot v}{x \cdot (y + z) \approx (x \cdot y) + (x \cdot z)}$	$\begin{array}{c c} + & a \quad b \\ \hline a & \\ b & \end{array}$	$\begin{array}{c c} \cdot & a \quad b \\ \hline a & \\ b & \end{array}$
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Find a two element **counterexample** to the following argument:

$\frac{fg(x) \approx gf(x)}{f(x) \approx g(x)}$	$\begin{array}{c c} f & \\ \hline a & \\ b & \end{array}$	$\begin{array}{c c} g & \\ \hline a & \\ b & \end{array}$
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Give a derivation using Birkhoff's Rules, stating reasons for your steps, to show that

$$x \cdot y \approx u \cdot v \vdash (x \cdot x) + (y \cdot y) \approx (y \cdot y) + (x \cdot x).$$

The following is a derivation:

1. $x \cdot y \approx u \cdot v$

Given

Fill in the reasons for the steps in the following derivation. [This is part of a derivation found by an automated theorem prover.]

1.	$x + 0$	\approx	x	given
2.	$x + (-x)$	\approx	0	given
3.	$x + y$	\approx	$y + x$	given
4.	$(x + y) + z$	\approx	$x + (y + z)$	given
5.	$x \cdot (y + z)$	\approx	$(x \cdot y) + (x \cdot z)$	given
6.	$(x + y) \cdot z$	\approx	$(x \cdot z) + (y \cdot z)$	given
7.	$x \cdot x$	\approx	x	given
8.	$(x \cdot y) + (x \cdot z)$	\approx	$x \cdot (y + z)$	
9.	$(x \cdot z) + (y \cdot z)$	\approx	$(x + y) \cdot z$	
10.	$(-x) + x$	\approx	$x + (-x)$	3 Subs
11.	$(-x) + x$	\approx	0	10, 2 Trans
12.	$(x + (-x)) + y$	\approx	$x + ((-x) + y)$	
13.	$x + ((-x) + y)$	\approx	$(x + (-x)) + y$	
14.	$(x + (-x)) + y$	\approx	$0 + y$	
15.	$0 + y$	\approx	$y + 0$	
16.	$y + 0$	\approx	y	
17.	$0 + y$	\approx	y	
18.	$(x + (-x)) + y$	\approx	y	
19.	$x + ((-x) + y)$	\approx	y	
20.	$((-x) + x) + y$	\approx	$(x + (-x)) + y$	
21.	$((-x) + x) + y$	\approx	y	
22.	$((-x) + x) + y$	\approx	$(-x) + (x + y)$	
23.	$(-x) + (x + y)$	\approx	$((-x) + x) + y$	
24.	$(-x) + (x + y)$	\approx	y	
25.	$y + x$	\approx	$x + y$	
26.	$(-x) + (y + x)$	\approx	$(-x) + (x + y)$	
27.	$(-x) + (y + x)$	\approx	y	
28.	$(x \cdot x) + (x \cdot y)$	\approx	$x \cdot (x + y)$	
29.	$(x \cdot x) + (x \cdot y)$	\approx	$x + (x \cdot y)$	