Name: ID:

PMath 330

Assignment 5

Mark

A Resolution Derivation

Given the collection of 8 clauses

1. $\{Q, S\}$ 2. $\{R, S\}$ 3. $\{\neg P, Q\}$ 4. $\{P, R\}$
5. $\{P, \neg Q\}$ 6. $\{\neg P, \neg R\}$ 7. $\{\neg Q, \neg S\}$ 8. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution derivation:

9. $\{Q, \neg R\}$ _____
10. $\{\neg Q, R\}$ _____
11. $\{Q, R\}$ _____
12. $\{\neg Q, \neg R\}$ _____
13. $\{Q\}$ _____
14. $\{\neg Q\}$ _____
15. $\{\ \}$ _____

Is it possible to find an assignment of truth values for the propositional variables P, Q, R, S that will satisfy the original eight clauses? (if yes, give one)

Given the collection $\mathcal{S}$ of 6 clauses

1. $\{P, \neg Q\}$ 2. $\{Q, S\}$ 3. $\{P, R\}$
4. $\{\neg P, \neg R\}$ 5. $\{\neg Q, \neg S\}$ 6. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution steps:

7. $\{Q, \neg R\}$ _____
8. $\{P, \neg S\}$ _____
9. $\{S, \neg S\}$ _____
10. $\{Q, \neg Q\}$ _____
11. $\{\neg Q, \neg R\}$ _____
12. $\{P, \neg P\}$ _____
13. $\{R, \neg R\}$ _____
14. $\{P, S\}$ _____
15. $\{P, \neg R\}$ _____
16. $\{P\}$ _____
17. $\{\neg R, S\}$ _____
18. $\{P, Q\}$ _____
19. $\{\neg R\}$ _____

Can you obtain any other clauses by resolution? _____

What does this say about the satisfiability of $\mathcal{S}$? _____

Apply the Davis-Putnam Procedure to the First Problem, showing just the $\mathcal{S}_i'$ and $\mathcal{U}_i$ steps (as done for the resolution on R below).

Resolution on R:

$\mathcal{S}_1'$: $\{Q, S\}$ $\overset{(1)}{\{R, S\}}$ $\{\neg P, Q\}$ $\overset{(2)}{\{P, R\}}$ $\{P, \neg Q\}$ $\overset{(3)}{\{\neg P, \neg R\}}$ $\{\neg Q, \neg S\}$ $\overset{(4)}{\{\neg R, \neg S\}}$

$\mathcal{U}_1$: $\overset{(1,3)}{\{\neg P, S\}}$ $\overset{(1,4)}{\{S, \neg S\}}$ $\overset{(2,3)}{\{P, \neg P\}}$ $\overset{(2,4)}{\{P, \neg S\}}$

Resolution on P:

$\mathcal{S}_2'$:

$\mathcal{U}_2$:

Resolution on Q:

$\mathcal{S}_3'$:

$\mathcal{U}_3$:

Resolution on S:

$\mathcal{S}_4'$:

$\mathcal{U}_4$:

Given the collection of five Horn clauses

1. $\{\neg P, Q\}$ 2. $\{P\}$ 3. $\{\neg R\}$ 4. $\{S, \neg Q\}$ 5. $\{\neg S, R\}$

find all clauses that can be derived using unit resolution:

	Clause	Reason
6.		
7.		
8.		
9.		
10.		
11.		
12.		

Given the clauses $\{\neg P, Q, R\}$ and $\{\neg Q, R, S\}$ prove, using just the definitions, that if $\vec{e}$ is a truth evaluation of P, Q, R, S that makes the two clauses true then it also makes the clause $\{\neg P, R, S\}$, obtained by resolving the two over Q, true.

Consider the propositional argument:

$$\begin{array}{ll} \mathsf{F}_1: & (P \to \neg Q) \to R \\ \mathsf{F}_2: & P \vee \neg (Q \to R) \\ \hline \mathsf{F}: & P \leftrightarrow (Q \leftrightarrow R) \end{array}$$

Give the conjunctive normal form for each of the following formulas:

F_1 : _____

F_2 : _____

$\neg\,\mathsf{F}$: _____

From this derive a set $\mathcal{S}$ of clauses such that $\mathsf{F}_1, \mathsf{F}_2 \ \therefore \mathsf{F}$ is valid iff $\neg\,\mathsf{Sat}(\mathcal{S})$. $\mathcal{S}$ has the clauses:

1. _____ 4. _____
2. _____ 5. _____
3. _____ 6. _____

Is $\mathcal{S}$ satisfiable? (Reasons)

Is the original argument valid? (Reasons)