Name: ID:

PMath 330

Assignment 5

Mark

A Resolution Derivation

Given the collection of 8 clauses

1. $\{Q, S\}$	2. $\{R, S\}$	3. $\{\neg P, Q\}$	4. $\{P, R\}$
5. $\{P, \neg Q\}$	6. $\{\neg P, \neg R\}$	7. $\{\neg Q, \neg S\}$	8. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution derivation:

9.	$\{Q, \neg R\}$	__________	13.	$\{Q\}$	__________
10.	$\{\neg Q, R\}$	__________	14.	$\{\neg Q\}$	__________
11.	$\{Q, R\}$	__________	15.	$\{\ \}$	__________
12.	$\{\neg Q, \neg R\}$	__________			

Is it possible to find an assignment of truth values for the propositional variables P, Q, R, S that will satisfy the original eight clauses? (if yes, give one)

Given the collection $\mathcal{S}$ of 6 clauses

1. $\{P, \neg Q\}$	2. $\{Q, S\}$	3. $\{P, R\}$
4. $\{\neg P, \neg R\}$	5. $\{\neg Q, \neg S\}$	6. $\{\neg R, \neg S\}$

fill in the reasons for the following resolution steps:

7.	$\{Q, \neg R\}$	__________	14.	$\{P, S\}$	__________
8.	$\{P, \neg S\}$	__________	15.	$\{P, \neg R\}$	__________
9.	$\{S, \neg S\}$	__________	16.	$\{P\}$	__________
10.	$\{Q, \neg Q\}$	__________	17.	$\{\neg R, S\}$	__________
11.	$\{\neg Q, \neg R\}$	__________	18.	$\{P, Q\}$	__________
12.	$\{P, \neg P\}$	__________	19.	$\{\neg R\}$	__________
13.	$\{R, \neg R\}$	__________			

Can you obtain any other clauses by resolution? __________

What does this say about the satisfiability of $\mathcal{S}$? ____________________

Apply the Davis-Putnam Procedure to the First Problem, showing just the ${\mathcal{S}_i}'$ and $\mathcal{U}_i$ steps (as done for the resolution on R below).

Resolution on R:

$${\mathcal{S}_1}': \quad \{Q, S\} \quad \overset{(1)}{\{R, S\}} \quad \{\neg P, Q\} \quad \overset{(2)}{\{P, R\}} \quad \{P, \neg Q\} \quad \overset{(3)}{\{\neg P, \neg R\}} \quad \{\neg Q, \neg S\} \quad \overset{(4)}{\{\neg R, \neg S\}}$$

$$\mathcal{U}_1: \quad \overset{(1,3)}{\{\neg P, S\}} \quad \overset{(1,4)}{\{S, \neg S\}} \quad \overset{(2,3)}{\{P, \neg P\}} \quad \overset{(2,4)}{\{P, \neg S\}}$$

Resolution on P:

${\mathcal{S}_2}'$:

$\mathcal{U}_2$:

Resolution on Q:

${\mathcal{S}_3}'$:

$\mathcal{U}_3$:

Resolution on S:

${\mathcal{S}_4}'$:

$\mathcal{U}_4$:

Given the collection of five Horn clauses

1. $\{\neg P, Q\}$ 2. $\{P\}$ 3. $\{\neg R\}$ 4. $\{S, \neg Q\}$ 5. $\{\neg S, R\}$

find all clauses that can be derived using unit resolution:

	Clause	Reason
6.	________	________
7.	________	________
8.	________	________
9.	________	________
10.	________	________
11.	________	________
12.	________	________

Given the clauses $\{\neg P, Q, R\}$ and $\{\neg Q, R, S\}$ prove, using just the definitions, that if $\vec{e}$ is a truth evaluation of P, Q, R, S that makes the two clauses true then it also makes the clause $\{\neg P, R, S\}$, obtained by resolving the two over Q, true.

Consider the propositional argument:

$$\begin{array}{ll} \mathsf{F}_1: & (P \to \neg Q) \to R \\ \mathsf{F}_2: & P \vee \neg(Q \to R) \\ \hline \mathsf{F}: & P \leftrightarrow (Q \leftrightarrow R) \end{array}$$

Give the conjunctive normal form for each of the following formulas:

F_1 : ____________________

F_2 : ____________________

$\neg\,\mathsf{F}$: ____________________

From this derive a set $\mathcal{S}$ of clauses such that $\mathsf{F}_1, \mathsf{F}_2 \;\therefore\; \mathsf{F}$ is valid iff $\neg\,\mathsf{Sat}(\mathcal{S})$. $\mathcal{S}$ has the clauses:

1. ____________ 4. ____________

2. ____________ 5. ____________

3. ____________ 6. ____________

Is $\mathcal{S}$ satisfiable? (Reasons)

Is the original argument valid? (Reasons)