Name: ID:

PMath 330

Assignment 4

Mark______

A Combined Truth Table

	P	Q	R	S	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12
1	1	1	1	1	0	1	0	1	0	0	1	0	0	1	1	1
2	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1	1
3	1	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0
4	1	1	0	0	0	0	1	1	1	1	0	0	0	1	1	0
5	1	0	1	1	1	0	0	0	0	1	0	1	0	1	1	0
6	1	0	1	0	1	0	1	0	0	1	0	1	0	1	1	0
7	1	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0
8	1	0	0	0	1	0	1	1	0	1	0	0	0	1	1	0
9	0	1	1	1	0	0	1	0	1	0	0	0	0	0	1	0
10	0	1	1	0	0	0	0	0	0	1	0	0	0	1	1	0
11	0	1	0	1	0	0	1	1	1	0	0	0	0	0	1	0
12	0	1	0	0	0	0	1	1	0	1	0	0	0	1	1	0
13	0	0	1	1	1	0	1	0	1	1	0	0	0	1	1	0
14	0	0	1	0	0	0	0	0	0	1	0	0	0	1	1	0
15	0	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0
16	0	0	0	0	0	0	1	1	0	1	0	0	0	1	1	0

- Which of the formulas F1–F12 are **truth equivalent**? ____________
- Which of the formulas F1–F12 are **tautologies**? ____________
- Which of the formulas F1–F12 are **contradictions**? ____________
- Determine if the following arguments are **valid**. If not, cite the number of a row of the truth table that refutes the argument.

  (a) F1,F5,F10 $\therefore$ F8 ANS____________ (b) F1,F3,F4,F5,F6,F7,F10 $\therefore$ F8 ANS____________
- Determine if the following collections of formulas are **satisfiable**. If so, cite the number of a row of the truth table that satisfies them.

  (a) F1,F5,F6 ANS____________ (b) F3,F4,F6,F8 ANS____________
- Find the **disjunctive normal form** of F1 (with respect to the variables P, Q, R, S).

  ____
- Find the **conjunctive normal form** of F10 (with respect to the variables P, Q, R, S).

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Translate the following argument (of Lewis Carroll) into propositional formulas. Then by using the premisses, or equivalent propositions, fill in the sequence of implications that shows the conclusion is valid.

1. No shark ever doubts that it is well fitted out.
2. A fish, that cannot dance a minuet, is contemptible.
3. No fish is quite certain that it is well fitted out, unless it has three rows of teeth,
4. All fishes, except sharks, are kind to children.
5. No heavy fish can dance a minuet.
6. A fish with three rows of teeth is to be respected.

Therefore, heavy fish are kind to children.

The universe of discourse is "fishes".
Use the following:

S: x is a shark
D: x is able to dance a minuet
F: x is certain that it is well fitted out
C: x is contemptible
T: x has three rows of teeth
H: x is heavy
K: x is kind to children

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

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$$H \to \_\_ \to \_\_ \to \_\_ \to \_\_ \to \_\_ \to K.$$

Find a **derivation** of the tautology

$$P \to (Q \to (P \to Q))$$

using the FŁ proof system. *[Hint: It can be done in 3 lines!]*

ANSWER:

	Formula	Reason
1.	__________	______
2.	__________	______
3.	__________	______

Here is a proof that **there is a derivation** of the same tautology. You may use, as reasons, any of the steps before Theorem D.0.19 in Appendix D.

				Reason
1.	P, Q	$\vdash$	$P \to (Q \to (P \to Q))$	D.0.18
2.	$P, \neg Q$	$\vdash$	$P \to (Q \to (P \to Q))$	______
3.	P	$\vdash$	$P \to (Q \to (P \to Q))$	______
4.	$\neg P, Q$	$\vdash$	$P \to (Q \to (P \to Q))$	______
5.	$\neg P, \neg Q$	$\vdash$	$P \to (Q \to (P \to Q))$	______
6.	$\neg P$	$\vdash$	$P \to (Q \to (P \to Q))$	______
7.		$\vdash$	$P \to (Q \to (P \to Q))$	______