Name: ID:

PMath 330

Assignment 3

Mark

Equivalent Formulas

In the following table you are asked to consider whether or not certain formulas are equivalent. The $\Box$s in the left column are to be replaced by the appropriate binary connective in each of the successive columns. For example, consider the row starting with $P \Box P \sim P$. Now go over to the column with the header $\wedge$. The question is whether or not $P \wedge P \sim P$. If the formulas are equivalent, put a check mark ($\surd$) in the box. Otherwise leave the box empty.

	$\vee$	$\wedge$	$\rightarrow$	$\leftrightarrow$	$\mid$	$\curlywedge$
$P \Box P \sim P$						
$P \Box (P \Box P) \sim (P \Box P) \Box P$						
$P \Box Q \sim Q \Box P$						
$P \Box (Q \Box P) \sim P$						
$P \Box (Q \Box R) \sim (P \Box Q) \Box R$						

Adequate Sets of Connectives

Circle (or highlight) the formulas among $0, 1, P, \neg P$ that can be represented by a formula $F(P)$ using (only) the connectives in $\mathcal{C}$:

Given				
$\mathcal{C} = \{\vee\}$	0	1	P	$\neg P$
$\mathcal{C} = \{\rightarrow\}$	0	1	P	$\neg P$
$\mathcal{C} = \{\leftrightarrow, \rightarrow\}$	0	1	P	$\neg P$
$\mathcal{C} = \{\wedge, 1\}$	0	1	P	$\neg P$

Circle (or highlight) the connectives that can be realized using the connectives in $\mathcal{C}$:

Given				
$\mathcal{C} = \{\rightarrow, \vee\}$	$\vee$	$\wedge$	$\rightarrow$	$\leftrightarrow$
$\mathcal{C} = \{\wedge, \leftrightarrow\}$	$\vee$	$\wedge$	$\rightarrow$	$\leftrightarrow$
$\mathcal{C} = \{\neg, \wedge\}$	$\vee$	$\wedge$	$\rightarrow$	$\leftrightarrow$
$\mathcal{C} = \{\rightarrow, \leftrightarrow\}$	$\vee$	$\wedge$	$\rightarrow$	$\leftrightarrow$

Substitution/Replacement

In each of the following inferences you are to choose the most inclusive answer for how the inference could be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1. $$\frac{\neg\neg P \ \sim\ P}{\neg\neg\neg P \ \sim\ \neg P}$$ ____________________

2. $$\frac{P \ \sim\ Q}{P \to P \ \sim\ Q \to Q}$$ ____________________

3. $$\frac{P \wedge \neg Q \ \sim\ \neg(P \to Q)}{Q \wedge \neg P \ \sim\ \neg(Q \to P)}$$ ____________________

4. $$\frac{P \vee Q \ \sim\ Q \vee P}{(P \vee Q) \to (P \vee Q) \ \sim\ (Q \vee P) \to (P \vee Q)}$$ ____________________

5. $$\frac{P \to (Q \to P) \ \sim\ 1}{(Q \to P) \to (P \to (Q \to P)) \ \sim\ 1}$$ ____________________

More on Adequate Connectives

Determine if the binary connective $\triangle$ defined by $P \triangle Q \ \sim\ P \wedge \neg Q$ is adequate. Give Reasons! [Either show that some adequate set of connectives can be expressed using $\triangle$, or prove that some propositional formula cannot be expressed using $\triangle$.]

Suppose you know that $\mathsf{F}(P, Q, R, S)$ and $\mathsf{G}(P, Q, R, S)$ are two propositional formulas such that $\mathsf{F} \wedge \mathsf{G}$ has the same truth table as $\mathsf{F} \vee \mathsf{G}$. Prove that F and G have the same truth tables.
PROOF:

Give an **inductive definition** of $Num(\mathsf{F})$, the number of occurrences of variables in a propositional formula F.