Name: ID:

PMath 330

Assignment 3

Mark________

**Equivalent Formulas**

In the following table you are asked to consider whether or not certain formulas are equivalent. The $\Box$s in the left column are to be replaced by the appropriate binary connective in each of the successive columns. For example, consider the row starting with $P \Box P \sim P$. Now go over to the column with the header $\wedge$. The question is whether or not $P \wedge P \sim P$. If the formulas are equivalent, put a check mark ($\surd$) in the box. Otherwise leave the box empty.

| | $\vee$ | $\wedge$ | $\rightarrow$ | $\leftrightarrow$ | $\mid$ | $\curlywedge$ |
|---|---|---|---|---|---|---|
| $P \Box P \sim P$ | | | | | | |
| $P \Box (P \Box P) \sim (P \Box P) \Box P$ | | | | | | |
| $P \Box Q \sim Q \Box P$ | | | | | | |
| $P \Box (Q \Box P) \sim P$ | | | | | | |
| $P \Box (Q \Box R) \sim (P \Box Q) \Box R)$ | | | | | | |

**Adequate Sets of Connectives**

Circle (or highlight) the formulas among $0, 1, P, \neg P$ that can be represented by a formula $F(P)$ using (only) the connectives in $\mathcal{C}$:

| Given | | | | |
|---|---|---|---|---|
| $\mathcal{C} = \{\vee\}$ | $0$ | $1$ | $P$ | $\neg P$ |
| $\mathcal{C} = \{\rightarrow\}$ | $0$ | $1$ | $P$ | $\neg P$ |
| $\mathcal{C} = \{\leftrightarrow, \rightarrow\}$ | $0$ | $1$ | $P$ | $\neg P$ |
| $\mathcal{C} = \{\wedge, 1\}$ | $0$ | $1$ | $P$ | $\neg P$ |

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Circle (or highlight) the connectives that can be realized using the connectives in $\mathcal{C}$:

| Given | | | | |
|---|---|---|---|---|
| $\mathcal{C} = \{\rightarrow, \vee\}$ | $\vee$ | $\wedge$ | $\rightarrow$ | $\leftrightarrow$ |
| $\mathcal{C} = \{\wedge, \leftrightarrow\}$ | $\vee$ | $\wedge$ | $\rightarrow$ | $\leftrightarrow$ |
| $\mathcal{C} = \{\neg, \wedge\}$ | $\vee$ | $\wedge$ | $\rightarrow$ | $\leftrightarrow$ |
| $\mathcal{C} = \{\rightarrow, \leftrightarrow\}$ | $\vee$ | $\wedge$ | $\rightarrow$ | $\leftrightarrow$ |

**Substitution/Replacement**

In each of the following inferences you are to choose the most inclusive answer for how the inference could be accomplished. The four choices are: **substitution**, **replacement**, **both**, **neither**.

1. $$\frac{\neg\neg P \;\sim\; P}{\neg\neg\neg P \;\sim\; \neg P}$$ ____________________

2. $$\frac{P \;\sim\; Q}{P \to P \;\sim\; Q \to Q}$$ ____________________

3. $$\frac{P \wedge \neg Q \;\sim\; \neg(P \to Q)}{Q \wedge \neg P \;\sim\; \neg(Q \to P)}$$ ____________________

4. $$\frac{P \vee Q \;\sim\; Q \vee P}{(P \vee Q) \to (P \vee Q) \;\sim\; (Q \vee P) \to (P \vee Q)}$$ ____________________

5. $$\frac{P \to (Q \to P) \;\sim\; 1}{(Q \to P) \to (P \to (Q \to P)) \;\sim\; 1}$$ ____________________

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**More on Adequate Connectives**

Determine if the binary connective $\triangle$ defined by $P \triangle Q \;\sim\; P \wedge \neg Q$ is adequate. Give Reasons! [Either show that some adequate set of connectives can be expressed using $\triangle$, or prove that some propositional formula cannot be expressed using $\triangle$.]

Suppose you know that $\mathsf{F}(P, Q, R, S)$ and $\mathsf{G}(P, Q, R, S)$ are two propositional formulas such that $\mathsf{F} \wedge \mathsf{G}$ has the same truth table as $\mathsf{F} \vee \mathsf{G}$. Prove that $\mathsf{F}$ and $\mathsf{G}$ have the same truth tables.
PROOF:

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Give an **inductive definition** of $Num(\mathsf{F})$, the number of occurrences of variables in a propositional formula $\mathsf{F}$.