Name: ID:

PMath 330

Assignment 2

Mark

The Expansion Theorem

Prove that if a term $F(X)$ in the language of classes is always equal to $AX \cup BX'$, where A and B are classes, then $A = F(1)$ and $B = F(0)$.
PROOF:

For the formula $F(A, B) = A \cup A'B$ carry out the following computations to calculate the expansion of $F(A, B)$ on A, B:

	Full Expression		Value
$F(1,1) =$		$=$	
$F(1,0) =$		$=$	
$F(0,1) =$		$=$	
$F(0,0) =$		$=$	

Thus expanding on A, B gives: $F(A, B) =$ ____________________________

For the formula $F(A, B, C) = (A \cup B)'(B \cup C)$ carry out the following computations to calculate the expansion of $F(A, B, C)$ on B:

	Full Expression		Simplified
$F(A,1,C) =$		$=$	
$F(A,0,C) =$		$=$	

Expanding on B gives: $F(A, B, C) =$ ____________________________

For the formula $F(A, B, C) = (A \cup B)(BC)'$ carry out the following computations to calculate the expansion of $F(A, B, C)$ on A, C:

	Full Expression		Simplified
$F(1, B, 1) =$		$=$	
$F(1, B, 0) =$		$=$	
$F(0, B, 1) =$		$=$	
$F(0, B, 0) =$		$=$	

Thus expanding on A, C gives: $F(A, B, C) =$ ____________________

Elimination

Given two classes A and B, prove that there is a class X such that $AX \cup BX' = 0$ holds if and only if $AB = 0$, that is, iff the intersection of A and B is empty.

PROOF:

For the formula $E(A,B,C) = (A \cup B \cup C)(A' \cup B' \cup C')$ carry out the following computations to eliminate B from the equation $E(A,B,C) = 0$:

	Full Expression		Simplified
$E(A,1,C) =$		$=$	
$E(A,0,C) =$		$=$	

Eliminating B gives (simplify first!): ________________ $= 0$

For the formula $E(A,B,C,D) = (A \cup B)(C \cup D)$ carry out the following computations to eliminate A, D from the equation $E(A,B,C,D) = 0$:

	Full Expression		Simplified
$E(1,B,C,1) =$		$=$	
$E(1,B,C,0) =$		$=$	
$E(0,B,C,1) =$		$=$	
$E(0,B,C,0) =$		$=$	

Eliminating A, D gives (simplify first!): ________________ $= 0$

Fill in the following tree to give a proof of the validity of the argument using the method of Lewis Carroll. Be sure to give the number of the reason for each boxed letter.

1 $C\,D\,E = 0$

2 $A\,B'\,C' = 0$

3 $D'\,E\,F' = 0$

4 $A'\,G\,E = 0$

5 $B'\,D'\,F = 0$

$B'\,G\,E = 0$