Preface

In the late 1980s Kevin Compton published papers showing how to apply an analysis of the popular partition identities to obtain monadic second-order limit laws, including 0–1 laws, for numerous classes of graphs, posets, etc. I was fascinated by this work. When he gave a colloquium talk in the early 1990s in Waterloo on his recent work on a limit law for Abelian groups, the temptation to learn more about this subject was irresistible. This engaged a favorite area of my own research, algebraic structures.

Compton’s papers can be somewhat opaque for specialists in combinatorics and number theory, as well as for specialists in logic, because of the intimate way he has woven these subjects together. After seeing the books [29], [30] of John Knopfmacher on abstract analytic number theory, it seemed worthwhile to separate Compton’s treatment into two parts, one on density in number systems, and the other on the application of number theoretic density results to obtain logical limit laws. Each of these parts is of interest in its own right.

The reader will find a leisurely and detailed exposition of Compton’s investigations, and closely related work of others, including recent contributions of Jason Bell, Edward Bender, Peter Cameron, Paweł Idziak, Arnold Knopfmacher, John Knopfmacher, Andrew Odlyzko, Bruce Richmond, András Sárközy, Cameron Stewart, Richard Warlimont, Alan Woods, and the author. The presentation is from the perspective of abstract number systems, in the spirit of John Knopfmacher’s work in abstract analytic number theory.

This book has been used as an undergraduate special topics reading text at the University of Waterloo. Part 1, on Additive Number Systems, is completely accessible to an advanced undergraduate student. All chapters preceding Chapter 6 are devoted to number theoretic density, requiring only the usual undergraduate background in analysis, especially in power series, and an exposure to abstract mathematics. The well known ratio test plays a central role. The section on asymptotics, at the end of Chapter 5, uses basic complex analysis, including the Cauchy integral formula. Chapter 6 covers the logical aspects for Part 1. This chapter is self-contained so that one can work through it without prior exposure to logic. It features one of the most delightful tools of logic, the Ehrenfeucht-Fraïssé games. (Having had a first course in logic, so that one is comfortable with first-order languages and structures, will no doubt make the chapter more rapid reading.)
Part 2, on Multiplicative Number Systems, offers the challenge and reward of becoming reasonably comfortable with Dirichlet series. The parallels with Part 1 show Dirichlet series as a natural companion of power series. Having worked through Part 1, one will be able to predict many of the results to be proved—the local density results of Part 1 seem, as if by magic, to reappear as global density results in Part 2. There is surely some deep connection between power series and Dirichlet series that we have not yet understood. The last chapter introduces the reader to the Feferman-Vaught Theorem, a favorite tool to analyze direct products, and Skolem’s analysis of first-order sentences about Boolean algebras.

The reader will find all the material needed to thoroughly understand the method of Compton for proving logical limit laws. Above all, I think one will be delighted to see so many interesting tools from elementary mathematics pull together to help answer the question “What is the probability that a randomly chosen structure has a given property?”

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For errata and updates see www.thoralf.uwaterloo.ca.

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