Lecture 5

In this lecture, we will learn about functions.

Definition A function is a rule that assigns to each element of one set exactly one element from another set.

notation $y = f(x)$

x = independent variable

y = dependent variable

The domain of a function is the set of all possible values that x can take. [input]

The range of a function is the set of all possible values that y can take. [output]

E.g.

Function	Domain	Range
$y = x^2$	$\mathbb{R}$ (all real no.s) or $(-\infty, \infty)$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$ (can't take $\sqrt{\cdot}$ of negative)	$[0, \infty)$
$y = \sin(x)$	$\mathbb{R}$	$[-1, 1]$

When finding the domain of a function, remember the following :-

1) No dividing by 0.

2) No square root $\sqrt{\ }$ (or even root $\sqrt[n]{\ }$, n even) of negative numbers.

3) No logarithm of a number ≤ 0.

e.g. Find the domain of $f(x)$.

1) $f(x) = \dfrac{3}{(x-1)(x-7)}$

Sol. The denominator can't be 0 $\Rightarrow$ $x-1 \neq 0$ and $x-7 \neq 0$ $\Rightarrow$ $x \neq 1$ and $x = 7$. All other values of x are allowed. Thus, domain is

$$\{x \in \mathbb{R} \mid x \neq 1, x \neq 7\} \text{ or } (-\infty, 1) \cup (1, 7) \cup (7, \infty).$$

2) $f(x) = \sqrt{x+3}$

Sol. we must have $x+3 \geq 0$ $\Rightarrow$ $x \geq -3$. Thus the domain is $\{x \in \mathbb{R} \mid x \geq -3\}$ or $[-3, \infty)$.

3) $f(x) = \dfrac{\sqrt{x^2-5x+6}}{x}$

Sol. We must have $x \neq 0$ and $x^2 - 5x + 6 \geq 0$

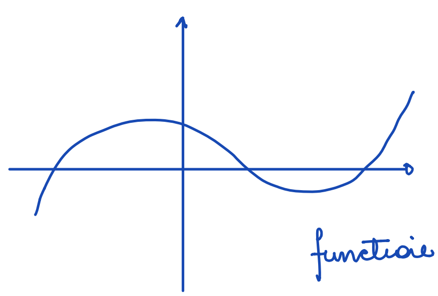
$\Rightarrow$ $x \neq 0$ and $(x-3)(x-2) \geq 0$

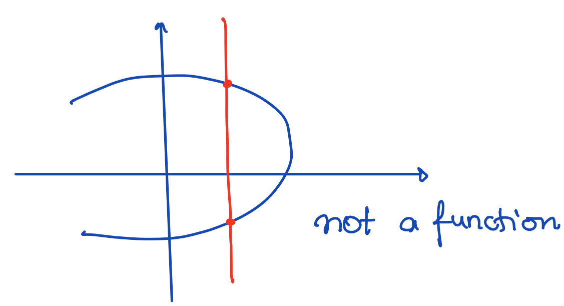
$\Rightarrow$ the domain is $(-\infty, 0) \cup (0, 2] \cup [3, \infty)$.

<u>Ques:</u> How do we know if an equation defines a function?

<u>Ans:</u>- We use the <u>vertical line test</u>.

<u>Vertical Line Test</u> :- If any vertical line intersects a graph at more than one point the it is NOT the graph of a function.





<u>Evaluating functions</u> : To evaluate $y = f(x)$ at a point $x = a$, simply replace every x in $f(x)$ by a.

e.g. If $f(x) = x^2 + \sqrt{x}$ then

- $f(0) = 0^2 + \sqrt{0} = 0$
- $f(1) = 1^2 + \sqrt{1} = 2$
- $f(-1)$ is not allowed as -1 is not in the domain.
- $f(x+h) = (x+h)^2 + \sqrt{x+h}$

Composition of functions :-

We can build new functions from old ones using composition.

The composition of functions f and g is the function $f \circ g$ which is defined by

$$(f \circ g)(x) = f(g(x))$$

(i.e., first apply g then apply f).

Similarly, we can define $g \circ f$ by

$$g \circ f(x) = g(f(x))$$

(first find $f(x)$ and then apply g to the result).

e.g. If $f(x) = x^2 + x$ and $g(x) = 5x - 2$, then

$$(f \circ g)(x) = f(g(x)) = f(5x-2) = (5x-2)^2 + (5x-2)$$

$$= 25x^2 + 4 - 10x + 5x - 2$$

$$= 25x^2 - 5x + 2$$

$$(g\circ f)(x) = g(x^2+x) = 5(x^2+x) - 2$$
$$= 5x^2 + 5x - 2$$

So, $$f\circ g\,(1) = 25.(1)^2 - 5.1 + 2 = 22$$
$$g\circ f\,(1) = 5.(1)^2 + 5.1 - 2 = 8$$

Warning: Usually $f\circ g \neq g\circ f$. This can be seen from the above example as well.