<u>Lecture 3</u>

Rational Inequalities

e.g. Solve $\dfrac{x+3}{x-7} > 2$.

- As in solving rational equations, move everything to the RHS and write as a single rational inequality.
- Find all values where the numerator or the denominator is 0.
- Plot these on a number line and check for points between these values. Include the interval if the inequa-lity is true.

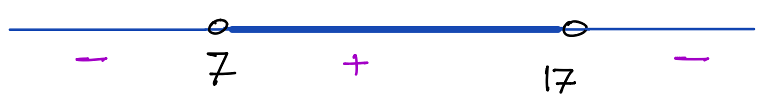
Let's see the example above.

$$\frac{x+3}{x-7} > 2 \Rightarrow \frac{x+3}{x-7} - 2 > 0$$

$$\Rightarrow \frac{x+3-2(x-7)}{x-7} > 0 \Rightarrow \frac{-x+17}{x-7} > 0$$

Numerator is 0 at $x = 17$, Denominator is 0 at $x = 7$

Test



Testing in the interval gives $x \in (7, 17)$. We use (,) to denote the exclusion of the points 7 and

17. Same reason for circles in the number line plot.

Exponents

Recall that for $n \geq 1$, $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n\text{-times}}$

$x^0 = 1$, $x^{-1} = \frac{1}{x}$ and $x^{-n} = \frac{1}{x^n}$.

Some properties

$a^m \cdot a^n = a^{m+n}$ $\qquad (a \cdot b)^m = a^m \cdot b^m$

$\frac{a^m}{a^n} = a^{m-n}$ $\qquad (a^m)^n = a^{m \cdot n}$

Remark :- $a^m + a^n \neq a^{m+n}$ or $a^m - a^n \neq a^{m-n}$

e.g. Simplify the following expressions :-

1) $2^7 \cdot 2^{11} = 2^{7+11} = 2^{18}$

2) $(3x)^3 = 3^3 \cdot x^3 = 27x^3$

3) $\left(\frac{x^2}{y^3}\right)^3 = \frac{x^{2 \cdot 3}}{y^{3 \cdot 3}} = \frac{x^6}{y^9}$

4) $\frac{a^{-3} b^5}{a^4 b^{-7}} = a^{-3-4} \cdot b^{5-(-7)} = a^{-7} b^{12} = \frac{b^{12}}{a^7}$

What about <u>fractional</u> exponents ?

Again, recall that $x^{1/n} = \sqrt[n]{x}$ (the n-th root of x)

and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

e.g.
- $9^{1/2} = \sqrt{9} = 3$
- $27^{1/3} = \sqrt[3]{27} = 3$
- $8^{2/3} = (8^{\frac{1}{3}})^2 = 2^2 = 4$

  or $= (8^2)^{1/3} = (64)^{1/3} = 4.$

<u>Radicals</u> [Roots]

<u>note</u> :- If n is an even natural number, then $\sqrt[n]{a}$ exists only for $a \geq 0$.

<u>Properties</u> :-
- $(\sqrt[n]{a})^n = a$ (if $\sqrt[n]{a}$ exists)
- $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$
- $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

- $\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n = \text{even} \\ a & \text{if } n = \text{odd} \end{cases}$

e.g. • $\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

- $\sqrt[3]{-27} = -3$
- $\sqrt{-16}$ does not exist.
- $\sqrt{(-3)^2} = |-3| = 3$ and $\sqrt[3]{-10^3} = -10$.

e.g. Simplify 1) $\sqrt[3]{48}$ 2) $\sqrt{x^3y^2}$ if $x, y \geq 0$

1) note : $48 = 8 \cdot 6 = 2^3 \cdot 6$

$\Rightarrow \sqrt[3]{48} = \sqrt[3]{2^3 \cdot 6} = 2 \cdot \sqrt[3]{6}$

2) note : $x^3 = x^2 \cdot x \Rightarrow \sqrt{x^3 \cdot y^2} = \sqrt{x \cdot x^2 \cdot y^2}$

$= xy\sqrt{x}$

<u>note</u> :- When simplifying a quotient, it helps to rationalize the denominator.

e.g. $\frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1^2-(\sqrt{2})^2} = -(1+\sqrt{2})$

Warning :- $\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$

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