Lecture 13

In the examples seen in Lec. 12 for calculating limits, we simply plugged in the value. However, that strategy doesn't always works.

<u>Ques</u> Calculate the following limits.

1) $\lim\limits_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$

2) $\lim\limits_{x \to \infty} \dfrac{2x^2 + 1}{3x^2 + 9x + 8}$

3) $\lim\limits_{x \to -\infty} \dfrac{2x + 1}{3x^2 + 9x + 2}$

4) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin(2x)}{\cos x}$

5) $\lim\limits_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

6) $\lim\limits_{x \to 0} \dfrac{|x|}{x}$

We can't simply plug in the values here as we will get what are known as "indeterminate forms". We need more strategies:-

Strategies for finding Limits

1) First try to plug $x = a$ and see if you get a finite value.

If you get, $\frac{c}{\pm\infty} = 0$, $\frac{c}{0} = \pm\infty$ and $c \cdot (\pm\infty) = \pm\infty$

where c is a finite constant number.

2) If 1) doesn't work then do we have an "indeterminate form" ?

$$\frac{0}{0}, \frac{\pm\infty}{\pm\infty}, \infty^0, 0^0, 1^\infty, 0\cdot\infty, \infty-\infty \text{ etc.}$$

If yes then try

- factoring and cancelling
- rationalizing the numerator or the denominator
- Using Trigonometric identities
- Calculating LHL and RHL and see if they are equal
- factoring the highest power of x in the numerator and denominator.

3) After applying any of the strategies in 2) we evaluate the limit again.

Let's try to solve the previous question.

1) $\lim\limits_{x\to 2} \dfrac{x^2-3x+2}{x-2}$

When we plug $x=2$, we get $\frac{0}{0}$, indeterminate form

We try to factorize and cancel

$$\lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}} = \lim_{x\to 2} x-1 = 2-1=1$$

2) $$\lim_{x\to\infty} \frac{2x^2+1}{3x^2+9x+8} = \frac{\infty}{\infty} \text{ form}$$

We factor x^2 from the numerator and the denom-
-inator to get

$$\lim_{x\to\infty} \frac{\cancel{x^2}\left(2+\frac{1}{x^2}\right)}{\cancel{x^2}\left(3+\frac{9}{x}+\frac{8}{x^2}\right)} = \lim_{x\to\infty} \frac{2+\overbrace{\frac{1}{x^2}}^{=0}}{3+\underbrace{\frac{9}{x}+\frac{8}{x^2}}_{=0}} = \frac{2}{3}$$

3) $$\lim_{x\to -\infty} \frac{2x+1}{3x^2+9x+2} = \lim_{x\to-\infty} \frac{\cancel{x}\left(2+\frac{1}{x}\right)}{\cancel{x^2}\,x\left(3+\frac{9}{x}+\frac{2}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{2}{3x+9} = \frac{2}{-\infty} = 0$$

4) $$\lim_{x\to \frac{\pi}{2}} \frac{\sin(2x)}{\cos x} = \frac{\sin\pi}{\cos\pi/2} = \frac{0}{0} \text{ form}$$

so we use trigonometric identity, $\sin 2\theta = 2\sin\theta\cos\theta$

to get $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{2\sin x \cancel{\cos x}}{\cancel{\cos x}} = \lim\limits_{x\to\frac{\pi}{2}} 2\sin x = 2\sin\frac{\pi}{2} = 2$

5) $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} = \dfrac{0}{0}$ form

We rationalize the numerator to get

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4} \frac{(\sqrt{x})^2 - 2^2}{(x-4)(\sqrt{x}+2)}$$

$$= \lim_{x\to 4} \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

6) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$. Here we'll calculate LHL and RHL.

Recall, $|x| = \begin{cases} -x, & x<0 \\ x, & x\geq 0 \end{cases}$

Thus $\lim\limits_{x\to 0^-} \dfrac{|x|}{x} = \dfrac{-x}{x} = -1 = \text{LHL}$

$\lim\limits_{x\to 0^+} \dfrac{|x|}{x} = \dfrac{x}{x} = 1 = \text{RHL}$

Thus LHL $\neq$ RHL so $\lim\limits_{x \to 0} \frac{|x|}{x}$ DNE.

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