Math 124 – Calculus & Vector Algebra for Kinesiology

Lecture 1

Welcome to Math 124! We will start this lecture with polynomials.

Definition A polynomial is a finite sum of terms where all variables have whole number exponents.

e.g. 1) $5x^3 + 3x^2 + 2x + 9$

exponent

x is the variable

Co-efficient of x^3

coefficient of x^2 ...

constant term

Operations on polynomials

1. Addition / Substraction :- Add / Substract the coefficients of the same power (or like terms).

e.g. $(5x^3 + 3x^2 + 2x + 9) + (2x^2 + 9x + 1)$

$= 5x^3 + 3x^2 + 2x^2 + 2x + 9x + 9 + 1 = 5x^3 + 5x^2 + 11x + 10$

$(5x^3 + 3x^2 + 2x + 9) - (2x^2 + 9x + 1)$

$= 5x^3 + (3x^2 - 2x^2) + (2x - 9x) + 9 - 1 = 5x^3 + x^2 - 7x + 8$

2. <u>Multiplication</u> :- Distribute and multiply each term and add the exponents.

e.g. 1) $(x+2)(x^2-x+3) = (x^3-x^2+3x) + (2x^2-2x+6)$

$= x^3+x^2+x+6$

2) $(x+1)^3 = (x+1)(x+1)(x+1)$

$= (x^2+x+x+1)(x+1)$

$= (x^2+2x+1)(x+1)$

$= (x^3+x^2)+(2x^2+2x)+(x+1)$

$= x^3+3x^2+3x+1$

[first multiply 1st and 2nd term and then multiply the result w/ the 3rd term]

Warning :- $(x+y)^n \neq x^n+y^n$ as can be seen from the above example.

<u>Factoring</u>

Factoring a polynomial is the <u>reverse</u> operation of multiplication. It involves breaking-up a polynomial into product of smaller polynomials.

First, check for <u>common factors</u> and factor them out.

e.g. $3x^2+6x = 3x(x+2)$

3 is common, x is common

<u>Factoring Quadratics</u> A quadratic polynomial is of the form $Ax^2 + Bx + C$, i.e., the highest exponent of x is 2.

For factoring quadratics, one can try to guess and check.

e.g. 1) $x^2 + 3x + 2$.

$A = 1$, $B = 3$, $C = 2$, so look for two numbers which add up to 3 and multiply to 2. A clear choice is 2 and 1 and hence $x^2 + 3x + 2 = (x+2)(x+1)$.

2) $x^2 + 2x + 1 = (x+1)(x+1)$

But it might not be easy to guess the factors everytime. In that case, we can use the

<u>Quadratic Formula</u>

The solutions to the quadratic $Ax^2 + Bx + C = 0$ are

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \text{provided } B^2 - 4AC \geq 0.$$

The two values of x, thus obtained are called roots.

Remark :- If $B^2 - 4AC < 0$ then the quadratic is called irreducible and it can't be factored.

So for factoring $Ax^2 + Bx + C$, we find the roots say r_1 and

r_2 and then $Ax^2+Bx+C = (x-r_1)(x-r_2)$.

e.g. 1) Factor $x^2+14x+24$.

$B^2-4AC = (14)^2-4\cdot 24 = 196-96 = 100 > 0 \Rightarrow$ can be factored.

The roots are $x = \dfrac{-14 \pm \sqrt{100}}{2} = \dfrac{-14 \pm 10}{2}$

$\Rightarrow x = -2$ or $x = -12$

so $x^2+14x+24 = (x+2)(x+12)$.

2) x^2+x+5

$B^2-4AC = 1-4\cdot 5 = -19 < 0 \Rightarrow$ the polynomial is irreducible.

Helpful Factorizations

1) Difference of squares : $x^2-y^2 = (x+y)(x-y)$

2) perfect square : $x^2+2xy+y^2 = (x+y)^2$

3) Difference of cubes : $x^3-y^3 = (x-y)(x^2+xy+y^2)$

4) Sum of cubes : $x^3+y^3 = (x+y)(x^2-xy+y^3)$

How to use the above ?

Ques:- Factorize $8x^3-1$.

Ans :- $8x^3-1 = (2x)^3 - 1^3 \Rightarrow$ difference of cubes

$\therefore \quad 8x^3-1 = (2x-1)\underbrace{(4x^2+2x+1)}$

can we factorize this more?

Here $B^2-4AC = 4 - 4\cdot4\cdot1 = -12 < 0 \Rightarrow$ irreducible.

Thus, $8x^3-1 = (2x-1)(4x^2+2x+1)$

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