Recognizing k-Clique Extendible Orderings

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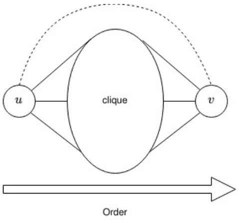

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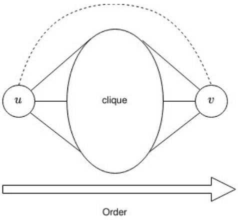

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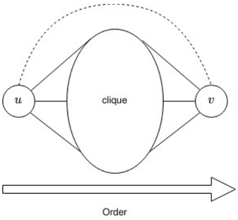

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It was observed that finding a maximum clique in a k-C-E graph can be done in time $n^{O(k)}$ (when given the ordering).

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- $n^{O(k)}$ algorithm for finding maximum clique is optimal assuming ETH (even when the ordering is given)
- If k is given as input, the problem is also coNP-hard.
- Verification problem is coNP-hard and W[1]-hard in general but FPT when treewidth is bounded.

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BETWEENNESS
Given: Universe U, family of triples $T = \{t_1, \ldots, t_m\}$ where each $t_i = (a_i, b_i, c_i)$ is an ordered triple of elements of U
Output: Does there exist an ordering of U such that for each $t_i \in T$, b_i comes between a_i and c_i in the ordering?

Reduction for $k \geq 4$

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"comes between" $:=$ either $a_i < b_i < c_i$ or $c_i < b_i < a_i$.

Known to be NP-hard.

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I.e. gadget will 'prune' out all the 'bad' orderings, keeping all the other orderings intact.

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- On the other hand, if there exists a valid betweenness ordering on U, then that ordering will carry over to the final graph G^*.

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Take disjoint union of G and F and then identify (contract) the vertices a with x, b with y, and c with z.

Since the new graph G' contains F as a subgraph, it must be the case that b comes between a and c in G' also.

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Separator Lemma

Let V_1, V_2 be a covering of $V(G)$ such that $S = V_1 \cap V_2$ is a separator. Given orderings σ_1 and σ_2 of V_1 and V_2 respectively, one can construct an ordering ϕ of $V_1 \cup V_2 = V(G)$, if the following conditions hold:

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Since $|S| = |V(G) \cap V(F)| = |\{a, b, c\}| = 3$, the lemma can be used only when $k \geq 4$. This is why we need a different reduction for $k = 3$.

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The Gadget: Takes in G and $a, b, c \in V(G)$. Takes the disjoint union of G and F and the identifies (contracts) the vertices a with x, b with y, and c with z.
Property of F: F has a k-C-E ordering and there exists $x, y, z \in V(F)$ such that in k-C-E ordering of F, y comes between x and z.

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Since F is a subgraph of G', and since either $x < y < z$ or $z < y < x$ must hold in F, this follows that either $a < b < c$ or $c < b < a$ holds in G'.

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Both properties of the gadget are satsified and we have our reduction.

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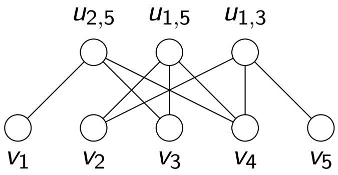

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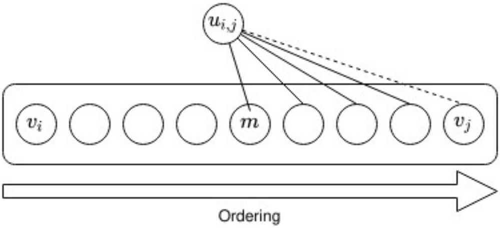
If $u_{i,j} < m$ then there exists an induced ordered K_{k+1}^{-} (See Figure).
Symmetrically, for $u_{i,j} > m$.



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- Can we find a $n^{O(k)}$ algorithm for maximum clique when the k-C-E ordering is *not* given?

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- Is the recognition problem FPT when paramaterized by treewidth?

Thank You