On the Parameterized Complexity of Deletion to $\mathcal{H}$-free Strong Components

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MFCS 2020

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Directed FVS and related problems

DIRECTED FEEDBACK VERTEX SET
Input: Directed graph D, integer k
Output: Does there exist a set S of size at most k such that $D - S$ is acyclic?

Best known FPT algorithm: $O^*(k!4^k)$ (here O^* notation suppresses polynomial factors)

Improving this is a big open problem in parameterized complexity.

Recent work by Göke et al. [CIAC 2019] designs FPT algorithms for related problems.

Directed FVS and related problems

1-Out Regular Vertex Deletion
Input: Directed graph D, integer k
Output: Does there exist a set S of size at most k such that every strong component of $D - S$ has every vertex of degree at most 1?

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Göke et al. gave a $2^{O(k^3)}n^{O(1)}$ algorithm for the first problem and a $4^k(ks+k+s)!n^{O(1)}$ algorithm for the second one.

Our problem

We generalize these problems to a more unified framework.

$\mathcal{H}$-FREE STRONG CONNECTED COMPONENT DELETION
Input: Directed graph D, integer k, finite family of graphs $\mathcal{H}$
Output: Does there exist a set S of at most k vertices such that every strong component of $D - S$ does not have a subgraph isomorphic to any graph $\mathcal{H}$

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Directed Feedback Vertex Set. Here $\mathcal{H}$ is an independent set on two vertices.

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- A $2^{O(k^3 \log k)} n^{O(h)}$ algorithm for when each graph in $\mathcal{H}$ has a 'rooted' property. Here h is the maximum size amognst all graphs in $\mathcal{H}$.

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Last two results improve on the bounds given by Göke et al.

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Algorithms for 1-Out Regular Vertex Deletion and Bounded Size Strong Component Vertex Deletion are based upon similar ideas.

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Symmetrically, there is a unique minimum 'furthest' separator i.e. separator C such that $R(S, C) \supseteq R(S, C')$ for all other minimum S-T separators C'.

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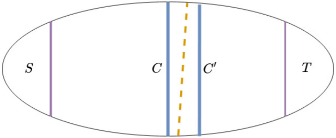

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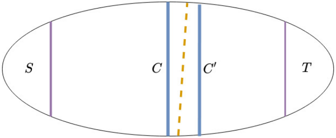

Lemma (Pushing Routine)

Given a minimum S-T separator C, in polynomial one can either

- Compute a minimum S-T separator C' that tightly covers C
- Conclude that there is no such C', i.e. C is the unique furthest minimum separator.

Important Separators

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For a graph D and subsets $S, T \subseteq V(G)$, an S-T separator C is said to be *important* if there is no other S-T separator C' such that $|C'| \leq |C|$ and $R(S, C') \supseteq R(S, C)$.

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Lemma

There are at most 4^k important separators of size at most k, and they can be enumerated in $O^*(4^k)$ time.

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Fundamental lemma for designing FPT algorithms for cut problems.

Prove something of the form "If there exists a solution, then there is a solution that contains an important separator", then branch on the 4^k important separators.

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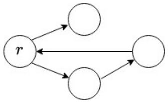

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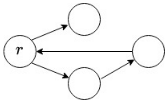

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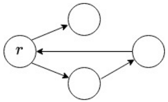

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We will look at the special case when each graph in $\mathcal{H}$ is rooted.

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The Problem

Rooted $\mathcal{H}$-free Strong Connected Component Deletion
Input: Graph D, integer k, finite family of graphs $\mathcal{H}$ where every graph is *rooted*
Output: Does there exist a set S of at most k vertices such that every strong component of $D - S$ does not have a subgraph isomorphic to any graph $\mathcal{H}$

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DISJOINT ROOTED $\mathcal{H}$-FREE SCC DELETION
Input: Graph D, integer k, finite family of graphs $\mathcal{H}$ where every graph is *rooted*, and solution $W \subseteq V(D)$ of size $k+1$
Output: Does there exist a solution of size k that is disjoint from W

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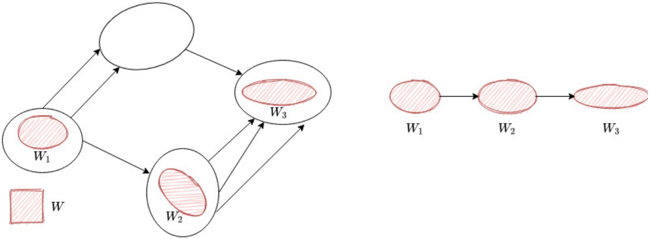

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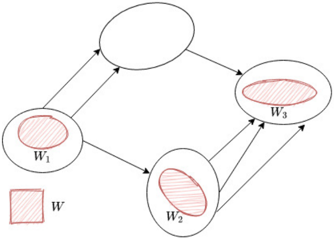

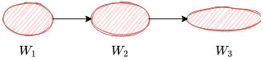

We start by *guessing* this ordered partition on W!

Now the problem reduces to the following: Given an ordered partition on $W = (W_1, \ldots, W_q)$, find a solution X that is disjoint from W and of size k such that the aforementioned topological ordering of $D - X$ induces same ordered partition on W.

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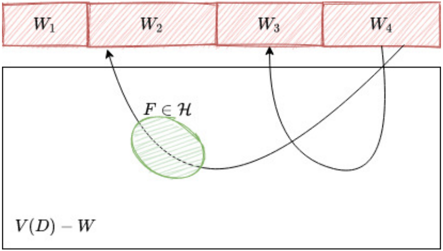

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At every branch, we either drop k or increase λ: the minimum S-T separator size.

Eventually when $\lambda = k$, either k must drop or we can conclude that is a NO-instance once $\lambda > k$, since every solution must contain an S-T separator.

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In the second case, the minimum S-T separator size λ increases because we add a vertex to T

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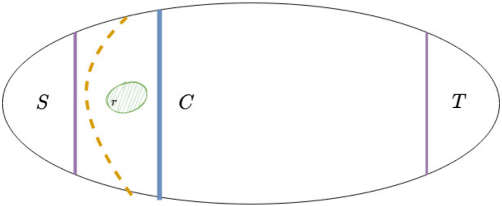

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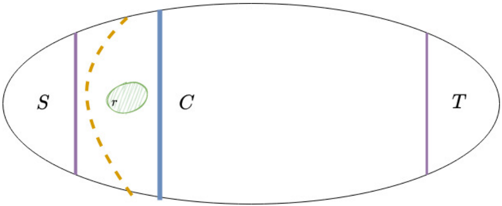

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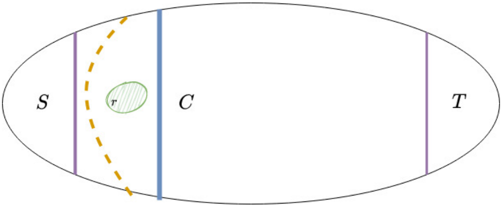

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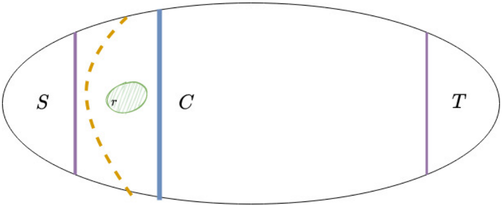

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Thus the minimum S-$(T \cup \{r\})$ separator size must be greater than λ.

Lemma (Pushing Routine)

Given a minimum S-T separator C, in polynomial one can either

- Compute a minimum S-T separator C' that tightly covers C
- Conclude that there is no such C', i.e. C is the unique furthest minimum separator.

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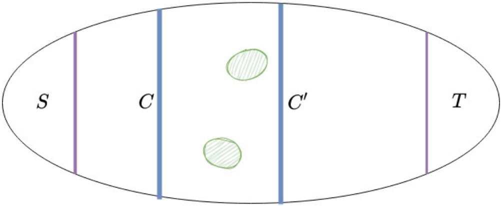

'Breaking' a forbidden subgraph F involves branching into the following cases:

1. Picking a vertex $v \in F$ into our solution
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Turns out by guessing which vertices of C that are reachable or unreachable in the final solution, we gain enough information to make progress in case 2 also.

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- What about infinite families?
- Recent result by Göke, Marx and Mnich [ICALP 2020] shows that one can design an FPT algorithm for when $\mathcal{H}$ is the set of cycles of length greater than some integer s.

Thank You