Pure Math 352, Midterm Information

Time: Friday, October 30, 4-6pm. **Place:** E2 (Engineering 2) 1303.

Topics: All of the course up to, and including the Maximum Modulus Principle, and Assignment #3. The exam will be roughly divided between

- definitions and theory from the lectures, summarised below; and
- (possibly modified versions of) questions from Assignments #1, #2 and #3.

List of Propositions, Lemmas and Theorems from Lectures

For any one of these results I may expect you to provide a proof. [Every theorem is based on a few simple ideas, some of which I indicate.]

- polar form; roots of unity
- Characterisation of $\mathbb{C}$-differentiability; Cauchy-Riemann equations.
- Power series lemma [comparison to geometric series].
- Differentiation of power series [good choice of $F : D(0,R)\times D(0,R) \to \mathbb{C}$; comparison with $\sum_n n\lambda^n$].
- Fundamental Theorem of Calculus (for $\mathbb{C}$-line integrals) [differentiate, use CR, appeal to usual FT of C].
- Estimation Lemma (for $\mathbb{C}$-line integrals)
- Cauchy-Goursat Theorem (for triangles) [contradiction argument: intersect nested triangles which violate to find point of non-differentiability: use FT of C + A1 Q3(a+b)].
- Existence of $\mathbb{C}$-primatives in star-like domains [$F(z) = \int_{[z_0,z]} f(w)dw$, use vanishing of triangle integrals to prove $F'(z) = f(z)$.]
- Cauchy's Theorem (in star-like domains) [Cauchy-Goursat + existence of primitives + FT of C].
- Cauchy integral formula [Cauchy's theorem + small circles].
- Holomorphicity $\Rightarrow$ analyticity [Cauchy integral formula + clever choice of series + exchange of limit and integral (uniform convergence on compact sets)].
- Morera's Theorem [existence of primitives + analyticity of primitive].
- Cauchy's Estimates [Cauchy integral formula + estimation lemma]
- Liouville's Theorem [Cauchy's estimates].
- Fundamental Theorem of Algebra [Liouville's theorem + limits at ∞].
- All derivatives vanish at a point $\Rightarrow$ constant (on connected domain) [set of all points at which all derivatives vanish has open relative complement & is itself open (power series)]
- Zero Lemma [power series starts at $N > 0$ + $f = g$ on $U \cap W \neq \varnothing$ implies $f = F|_U$, $g = F|_V$ for F holomorphic on $U \cap V$].
- Classification of Singularities [removable: clever choice of g + zero lemma $\Rightarrow$ function can be extended to holomorphic function over singularity; pole: local reciprocal to function in punctured neighbourhood + zero lemma + reciprocate back + analyticity; essential: range not dense implies either pole or removable]
- Zero set has cluster point $\Rightarrow$ constant [Zero lemma].
- Maximum Modulus Priciple [Parseval identity + estimates with integrals].