Pure Math 351, Final Review

Final exam time: December 7, 7:30-10PM, **Room:** MC4020.

Topics from lectures

Be expected to give an accurate definition of any word/phrase, used in the descriptions below.

Expect to know how a result is proven, unless otherwise indicated.

• Continuous functions take compact sets to compact sets; Extreme Value Theorem.

• Characterizations of compactness: every sequence admits converging subsequence; total boundedness and completeness.

• Characterizations of relative compactness for a subset of a complete space: every sequence admits Cauchy subsequence; total boundedness

• Equivalence of norms on $\mathbb{R}^n$.

• A linear map between normed vector spaces is continuous if and only if it is Lipschitz.

• Contraction Mapping Theorem.

• Picard-Lindelöf Theorem (case where $(b-a)L<1$, only).

• Edelstein's Theorem (existence and uniqueness of fixed point, only).

• Baire Category Thoerem, for a complete metric space: (i) intersection of countable family of open dense subsets is dense; (ii) a meager set has empty interior.

• $\mathbb{R}\setminus\mathbb{Q}$ is G_δ, hence $\mathbb{Q}$ is not G_δ.

• Uniform Boundedness Principle.

• Banach-Steinhaus Theorem.

• A Baire-1 function admits a point of continuity in any open interval (Statement only).

• A Baire-1 function is continuous on a dense G_δ set.

• There is no differentiable function $f:(0,1)\to\mathbb{R}$ whose derivative is continuous exactly on $(0,1)\cap\mathbb{Q}$.

• There exists a sequence $(q_n)_{n=1}^\infty$ of symmetric polynomials for which (i) $q_n\geq 0$ on $[-1,1]$; (ii) $\int_{-1}^1 q_n=1$; (iii) for $\delta>0$, $\lim_{n\to\infty}\left(\int_{-1}^{-\delta}+\int_\delta^1\right)q_n=0$. (Statement only.)

• Weierstrauss Approximation Theorem (applied to $f\in C[0,1]$ with $f(0)=0=f(1)$, only).

• Separability of $C[0,1]$.

- Stone's Approximation Theorem (statement only).
- Stone-Weierstrauss Theorem.
- Stone-Weierstrauss Theorem (Corollary) for point separating subalgebras of $C_{x_0}(X)$.
- Equicontinuity on a compact domain implies uniform equicontinuity
- Arzela-Ascoli Theorem.

Relevant assignment questions.

I may ask for simplified variants, rather than the questions as literally stated on the assignments. I will not ask questions related to the omitted assignment questions, except for the aspects of their results which pertain to the included material.

A3: Q4 (a), (b).

A5: Q1 (a), (b) (may pick other examples), (c), (d); Q2 (a) (Show that there is a sequence $(x_n)_{n=1}^{\infty}$ with $n_1 < n_2 < \dots$ for which $X = \bigcup_{j=1}^{n_k} B[x_j, \frac{1}{k}]$ for each k), (c) (Only part of this. Suppose we have a Cauchy sequence $(f_n)_{n=1}^{\infty} \subseteq C(C, Y)$ for which $f_n(C) \subseteq f_{n+1}(C)$ and $\bigcup_{n=1}^{\infty} f_n(C)$ is dense in Y. Then there exits a continuous surjection f in $C(C, Y)$.); Q3 (a), (b) (square roots only); Q4.

A6: Q1 (a), (b), (c), (d); Q2 (a); Q3 (a), (b), (c); Q4 (a), (b); Q5; Q6 (b).

A7: Q1 (a), (b), (d); Q2; Q3 (b), (c).