

**PMATH 965 -- Topics in Geometry and Topology:  
Geometric invariant theory, symplectic reduction and moduli spaces  
Course outline – Winter 2021**

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*Instructor:* Ruxandra Moraru

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*Virtual lectures:* MWF 10:00-10:50 (the lectures will be delivered live via zoom).

*Virtual Office hours:* Th 10:00--11:30, or by appointment.

*Registration:* [Zoom link for lectures and office hours](#).

*Course webpage:* Can be found at <https://learn.uwaterloo.ca>. This page contains course handouts (assignments and other), and is used to make announcements to the class. It is important that you register to the site and enter your email address to receive announcements.

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*Overview:* Many important examples of topological spaces, manifolds and varieties are constructed as quotients. Geometric invariant theory (GIT) and symplectic reduction provide methods for constructing quotients by groups actions in algebraic and symplectic geometry, respectively. In particular, moduli spaces of bundles or varieties are often obtained this way. This course will be an introduction to GIT and symplectic reduction with an application of these techniques to certain moduli problems.

*Outline of topics:* Algebraic groups and Lie groups; group actions and quotients; examples of affine and projective GIT quotients; criteria for (semi)stability; overview of sheaf cohomology; line bundles and divisors on curves; vector bundles and locally free sheaves; moduli of vector bundles on curves; symplectic structures; Hamiltonian actions, moment maps, and symplectic quotients; examples; GIT vs symplectic quotients; moduli spaces of bundles/connections as symplectic quotients (time permitting)

*Prerequisites:* The course should be accessible to students who have taken PMATH 465 (Geometry of Manifolds), PMATH 764 (Algebraic Geometry) or an equivalent course.

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*Suggested texts:*

1. N. R. Wallach, *Geometric Invariant Theory. Over the Real and Complex Numbers*, Universitext, Springer, 2017 (available online through the University of Waterloo library)
2. M. Brassil, *Geometric Invariant Theory*, 2012
3. I. Dolgachev, *Lectures on Invariant Theory*, 2003 (available online through ResearchGate)
4. A. Cannas da Silva, *Lectures on Symplectic Geometry*, Lecture Notes in Mathematics: 1764, Springer, 2008 (available online through the University of Waterloo library)
5. J. M. Lee, *Introduction to Smooth Manifolds*, Graduate Texts in Mathematics: 218, Springer, 2012 (available online through the University of Waterloo library)
6. S. Kobayashi, *Differential Geometry of Complex Vector Bundles*, Princeton, 1987
7. D. Huybrechts, *Complex Geometry: An Introduction*, Universitext, Springer, 2004 (available online through the University of Waterloo library)

*Additional references:*

1. D. Mumford, J. Fogarty and F. Kirwan, *Geometric Invariant Theory*, Ergebnisse der Mathematik und ihrer Grenzgebiete: 34, Springer, 3<sup>rd</sup> enlarged edition, 2003
2. A. H. W. Schmitt, *Geometric Invariant Theory and Decorated Principal Bundles*, Zurich Lectures in Advanced Mathematics, European Mathematical Society, 2008
3. R. Thomas, *Notes on GIT and symplectic reduction for bundles and varieties*, [arXiv:math/0512411v3](https://arxiv.org/abs/math/0512411v3)
4. V. Hoskins, *Moduli problems and geometric invariant theory*

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*Method of evaluation:* Your final grade will be based on 3 assignments, which will be posted on the course webpage and distributed via crowdmark. You can also get extra credit for typesetting the course notes via Overleaf.

*(Tentative) schedule of assignments:*

- Sunday, 7 Feb: Assignment 1
- Sunday, 7 Mar: Assignment 2
- Wednesday, 14 Apr: Assignment 3

*Academic Integrity:* In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check <http://www.uwaterloo.ca/academicintegrity/> for more information.]

*Grievance:* A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

*Discipline:* A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>. For typical penalties check Guidelines for the Assessment of Penalties, <http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm>.

*Appeals:* A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>.

*Note for students with disabilities:* The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability,

please register with the OPD at the beginning of each academic term.

*Mental Health Support:* The Faculty of Math encourages students to seek out mental health support if needed.

On-campus Resources:

- Campus Wellness <https://uwaterloo.ca/campus-wellness/>
- Counselling Services: [counselling.services@uwaterloo.ca/](mailto:counselling.services@uwaterloo.ca) 519-888-4567 ext 32655
- MATES: one-to-one peer support program offered by Federation of Students (FEDS) and Counselling Services: [mates@uwaterloo.ca](mailto:mates@uwaterloo.ca)
- Health Services: located across the creek from the Student Life Centre, 519-888-4096.

Off-campus Resources:

- Good2Talk (24/7): Free confidential help line for post-secondary students. Phone: 1-866-925-5454
- Here 24/7: Mental Health and Crisis Service Team. Phone: 1-844-437-3247
- OK2BME: set of support services for lesbian, gay, bisexual, transgender or questioning teens in Waterloo. Phone: 519-884-0000 extension 213

*Diversity:* It is our intent that students from all diverse backgrounds and perspectives be well served by this course, and that students' learning needs be addressed both in and out of class. We recognize the immense value of the diversity in identities, perspectives, and contributions that students bring, and the benefit it has on our educational environment. Your suggestions are encouraged and appreciated. Please let us know ways to improve the effectiveness of the course for you personally or for other students or student groups. In particular:

- We will gladly honour your request to address you by an alternate/preferred name or gender pronoun. Please advise us of this preference early in the semester so we may make appropriate changes to our records.
- We will honour your religious holidays and celebrations. Please inform of us these at the start of the course.
- We will follow AccessAbility Services guidelines and protocols on how to best support students with different learning needs.