WaterColor Plenary Abstracts

Dan Cranston (Virginia Commonwealth University)

Title: Using the Potential Method to Color Near-Bipartite Graphs

Abstract: A graph G is \emph{near-bipartite} if we can partition $V(G)$ as (I,F) where I is an independent set and F induces a forest. Similar to the problem of 3-coloring, deciding whether a graph is near-bipartite is NP-hard. Thus, we seek sufficient conditions. We show that a multigraph G is near-bipartite if $3|W|-2|E(G[W])|\ge -1$ for every $W\subseteq V(G)$, and G contains no K_4 and no Moser spindle. We show that a simple graph G is near-bipartite if $8|W|-5|E(G[W])|\ge -4$ for every $W\subseteq V(G)$ and G contains no subgraph in some finite family (each member of which is not near-bipartite). Each result is very sharp. Namely, if we weaken the lower bound in either hypothesis by 1, then the resulting statement is false infinitely often.

Both results are proved using the potential method, a powerful technique for coloring sparse graphs. As a consequence, both theorems lead to polynomial-time algorithms to find the desired partition.

This is joint work with Matthew Yancey.

Maria Chudnovsky (Princeton University)

TITLE: Minimal separators in even-hole-free graphs

ABSTRACT: It is not known whether there is a polynomial time algorithm to find the maximum size (or weight) of a stable set in an even-hole-free graph. A "pyramid" in a graph is an induced subgraph that consists of a triangle and a vertex with three vertex-disjoint paths to it. It seems that if an even-hole-free graph contains a pyramid, then its structure is fairly tightly organized around the pyramid, and this structure may be exploited to solve the stable set problem. But what about even-hole-free graphs without pyramids? A different idea is needed there. Recently a new powerful method of "potential maximal cliques" was introduced. Using this method one can solve the maximum stable set problem for every class of graphs all of whose members have polynomially many minimal separators. It turns out

that this approach works for the class of even-hole-free graphs that do not contain pyramids.
In joint work with Nicolas Trotignon, Stephan Thomasse and Kristina Vuskovic we were able to show that if G is an n-vertex graph with no even-hole and no pyramid, then G has at most $4n^12$ minimal separators.(In fact, we can relax the assumptions a little bit further). In this talk we will discuss the main ideas of the proof.

Louis Esperet (Universite Grenoble Alpes)

Title: Distributed graph coloring

Abstract: In this talk I will review old and new techniques in the design of efficient distributed algorithms for the graph coloring problem.

Ken-ichi Kawarabayashi (National Institute of Informatics)

Title: Some algorithmic aspect using the Four Color Theorem

Abstract: In this talk, we will survey some recent work for the following.

1. If a given graph contains a ``huge'' planar induced subgraph, then we can detect an approximately minimum hitting set, and hence we can use this set to color this graph.

2. If a given graph contains a ``huge'' induced subgraph that can be embedded in a surface, then we can detect an approximately minimum hitting set, and hence we can embed almost all vertices in a surface to color vertices.

In addition, we report some ``unrelated'' topic on Kempe chains, saying that they do not care about history.

Sophie Spirkl (Princeton University)

Title: List-three-coloring $(P_6 + rP_3)$-free graphs

Abstract: I will talk about a polynomial-time algorithm for list-three-coloring graphs that do not contain the disjoint union of a six-vertex path and r disjoint three-vertex paths as an induced subgraph.

Joint work with Maria Chudnovsky, Shenwei Huang, and Mingxian Zhong.

Alex Scott (Oxford University)

Title: Holes in graphs of large chromatic number

Abstract: If a graph G has large chromatic number, then what can we say about its induced subgraphs? In particular, if G does not contain a large clique, then what holes must it contain? Thirty years ago, Andras Gyarfas made a sequence of beautiful conjectures on this topic. We will discuss the recent resolution of these conjectures, and other related results.

Joint work with Maria Chudnovsky, Paul Seymour and Sophie Spirkl.

Paul Seymour (Princeton University)

Title: Short directed cycles in bipartite digraphs

Abstract: The Caccetta-Haggkvist conjecture implies that for every integer $k>0$, if G is a bipartite digraph, with n vertices in each part, and every vertex has out-degree more than $n/(k+1)$, then G has a directed cycle of length at most 2k. If true this is best possible, and we have proved it for k = 1,2,3,4,6 and all $k>224538$. The talk will survey these results and some related material.

Joint work with Sophie Spirkl.

Carsten Thomassen (Technical University of Denmark)

Title: Countable weighted graphs with no unfriendly partitions.

Abstract: The vertex set of every finite graph can be partitioned into two sets such that each vertex has at least as many vertices in the opposite set as in its own set. Such a partition is called *unfriendly.* It is known that there are infinite graphs with no unfriendly partition but it is open if every countable graph has an unfriendly partition. In this talk we show that the there are edge-weighted countable graphs with no unfriendly partition.

David Wood (Monash University)

Title: The product structure of graph classes

Abstract: We prove that every planar graph is a subgraph of the strong product of a path with some graph of bounded treewidth. Analogous theorems are obtained for graphs of bounded Euler genus and graphs excluding a fixed minor, as well as for certain non-minor-closed classes, such as graphs that can be drawn with a bounded number of crossings per edge. These results leads to proofs of the conjecture of Heath, Leighton and Rosenberg (1992) that planar graphs have bounded queue-number, and the conjecture of Alon, Grytczuk, Haluszczak and Riordan (2002) that planar graphs have bounded nonrepetitive chromatic number. We also obtain a simple proof of the result by DeVos, Ding, Oporowski, Sanders, Reed, Seymour and Vertigan (2004), which says that graphs excluding a fixed minor have low treewidth colourings.

This is joint work with Vida Dujmovi\'c, Gwena\"el Joret, Piotr Micek, Pat Morin and Torsten Ueckerdt [arXiv:1904.04791] and Vida Dujmovi\'c, Louis Esperet, Gwena\"el Joret and Bartosz Walczak [arXiv:1904.05269].

Xuding Zhu (Zheijiang Normal University)

Title: The Alon-Tarsi number and defective painting game on planar graphs

Abstract: It was proved independently by Eaton and Hull, and Skrekovski that if ˇ L is a 3-list assignment of a planar graph G, then there is a subgraph H of G of maximum degree 2 such that G − E(H) is L-colourable, and proved by Cushing and Kierstead that if L is a 4-assignment of a planar graph G, then G has a subgraph H of maximum degree 1 such that G − E(H) is L-colourable. We are interested in the problem whether we can find such a subgraph H that works for all list assignment L. In other words, we are interested in the following problems: (1) Is it true every planar graph G has a subgraph H of maximum degree 2 such that G − E(H) is 3-choosable? (2) Is it true that every planar graph G has a matching M such that G − M is 4-choosable? It turns out that the answers to (1) is negative, and the answer to (2) is positive. We show that there is a planar graph G such that for any subgraph H of maximum degree 3, G − E(H) is not 3-choosable. On the other hand, every planar graph G has a matching M

such that $G - M$ is 4-choosable. The latter result is proved by showing that $G - M$ has Alon-Tarsi number at most 4. This implies that an on-line version of Cushing-Kierstead's result is true. The on-line version of defective list colouring is defined through a two person game. Assume G is a graph, d, k are non-negative integers. A d-defective kpainting game on G is played by two players: Lister and Painter. Initially, each vertex has k tokens and is uncoloured. In each round, Lister chooses a set M of uncoloured vertices, and removes one token from each vertex in M. Painter chooses a subset X of M that induces a subgraph G[X] of maximum degree at most d. The vertices in X are considered to have been coloured in this round. If at the end of some round, an uncoloured vertex has no token left, then Lister wins the game. Otherwise at the end of some round, all vertices are coloured, Painter wins the game. We say G is d-defective k-paintable if Painter has a winning strategy in this game. It follows from the definition that d-defective k-paintable implies d-defective k-choosable. The converse is not true. We proved that there are planar 1 graphs that are not 2-defective 3-paintable. On the other hand, the above result on Alon-Tarsi number of $G - M$ implies that every planar graph is 1-defective 4-paintable.

This talk contains joint work with Ringi Kim, Seog-Jin Kim, Grzegorz Gutowski, Ming Han, Tomasz Krawczyk and Jaroslaw Grytczuk.