WaterColor Contributed Abstracts

Speaker: Amena Assem (University of Waterloo)

Title: Planar Plus Two Edges is 5-Choosable

Abstract: We show that graphs that can be obtained from a planar graph by adding two edges are 5-choosable. This is the maximum number of edges that can be added without losing 5-choosability. For example, K6 is three edges far from planar but has chromatic number 6. To arrive at this, we proved an extension of a theorem of Thomassen and an extension of a theorem of Postle and Thomas. The difference between our theorems and those of Thomassen and of Postle and Thomas is that we allow the graph to contain an inner 4-list vertex. We also use a colouring technique from two papers by Dvoˇr´ak, Lidick´y and Skrekovski, and independently by Compos and Havet.

This result was the subject of my masters thesis under the supervision of Bruce Richter.

Speaker: Anton Bernshteyn (Carnegie Mellon University)

Title: Equitable colourings of infinite graphs

Abstract: A proper k-colouring of a finite graph G is called equitable if every two colour classes differ in size at most by one. In particular, if G has n vertices and k divides n, then in an equitable k-colouring of G every colour class has size exactly n/k. There is a natural way to extend this definition to infinite graphs on probability spaces. Namely, if G is a graph whose vertex set $V(G)$ is a probability space, then a proper k-colouring of G is equitable when every colour class has measure $1/k$. In this talk I will discuss extensions of some classical results about equitable colourings to this setting, including an infinite version of the Hajnal--Szemer\'{e}di theorem on equitable k-colourings for $k \geq \Delta(G)+1$.

This is joint work with Clinton Conley.

Speaker: Rutger Campbell (University of Waterloo)

Title: Hajós construction for digraphs

Abstract: We give a method to construct a family of digraphs that cannot be (*k-1)*-coloured and which contains all *k*-critical digraphs. We also look at an application of this construction.

Speaker: Kathie Cameron (Wilfred Laurier University)

TItle: (Cap, Even Hole)-Free Graphs: Colouring, χ(G)-Boundedness and Hadwiger’s Conjecture

Abstract: A hole is a chordless cycle with at least 4 vertices, and is even if it has an even number of vertices. A cap consists of a hole together with an additional vertex which creates a triangle with the hole. A graph is (cap, even hole)-free if it has no induced cap or even hole. We give an explicit construction of (cap, even-hole)-free graphs. Using this, we prove that every such graph G has a vertex of degree at most 3 2 ω(G) − 1, and hence χ(G) ≤ 3 2 ω(G), where ω(G) denotes the size of the largest clique in G and χ(G) denotes the chromatic number of G. We give polynomial-time algorithms for minimum colouring and maximum weight stable set in these graphs. Our algorithms are based our results that (triangle, even hole)-free graphs have treewidth at most 5 and that (cap, even hole)-free graphs without clique cutsets have clique-width at most 48. A minor of a graph G is obtained from a subgraph of G by contracting edges. In 1943, Hadwiger made his famous conjecture: For every integer t ≥ 0, every graph with no Kt+1 minor is t-colourable. Hadwiger proved the conjecture for t = 3. For t = 4, it is equivalent to the Four Colour Theorem. Robertson, Seymour and Thomas proved it for t=5, using the 4CT. For t ≥ 6, it remains open. We prove Hadwiger’s Conjecture for (cap, even hole)-free graphs, and for some related classes of graphs.

This is joint work with Kristina Vu˘skovi´c, and parts are joint with Murilo V. G. da Silva and Shenwei Huang.

Speaker: Ilkyoo Choi (Hankuk University of Foreign Studies)

Title: Degeneracy and Colorings of Squares of Planar Graphs without 4-Cycles

Abstract: We prove several results on coloring squares of planar graphs without 4-cycles. First, we show that if G is such a graph, then G^2 is $(\Delta(G)+72)$-degenerate. This implies an upper bound of $\Delta(G)+73$ on the chromatic number of G^2 as well as on several variants of the chromatic number such as the list-chromatic number, paint number, Alon--Tarsi number, and correspondence chromatic number. We also show that if $\Delta(G)$ is sufficiently large, then the upper bounds on each of these parameters of G^2 can all be lowered to $\Delta(G)+2$ (which is best possible). To complement these results, we show that 4-cycles are unique in having this property. Specifically, let S be a finite list of positive integers, with $4\notin S$. For each constant C, we construct a planar graph $G_{S,C}$ with no cycle with length in S, but for which $\chi(G_{S,C}^2) > \Delta(G_{S,C})+C$.

Speaker: Linda Cook (Princeton University)

Title: Detecting a Long Even Hole

Abstract: In 1991, Bienstock showed that it is NP-Hard to test whether a graph G
has an even hole containing a specified vertex v in G. In 2002,
Conforti, Cornuéjols, Kapoor and Vušković gave a polynomial-time

algorithm to test whether a graph contains an even hole using a
structure theorem about even-hole-free graphs from an earlier paper.
In 2003, Chudnovsky, Kawarabayashi and Seymour provided a simpler
polynomial time algorithm that searches for even holes directly. We
extend this result by presenting a polynomial time algorithm to
determine whether a graph has an even hole of length at least k for a
given $k \geq 4$.

Joint work with Paul Seymour.

Speaker: James Davies (University of Waterloo)

Title: Circle graphs are polynomially χ-bounded

Abstract: We prove a polynomial χ-bounding function for circle graphs.

Joint work with Rose McCarty.

Speaker: Cemil Dibek (Princeton University)

Title: Strongly Perfect Claw-free Graphs - A Short Proof

Abstract: A graph is strongly perfect if every induced subgraph H has a stable set that meets every maximal clique of H. A graph is claw-free if no vertex has three pairwise non-adjacent neighbors. The characterization of claw-free graphs that are strongly perfect by a set of forbidden induced subgraphs was conjectured by Ravindra [1] in 1990 and proved by Wang [2] in 2006. Here we give a short proof of this characterization.

Joint work with Maria Chudnovsky and Andrei Graur.

[1] G. Ravindra. “Research problems”, Discrete Mathematics, 80 (1990), 105-107.
[2] H.Y. Wang. “Which claw-free graphs are strongly perfect?”, Discrete Mathematics, 306 (19-20) (2006), 2602–2629.

Speaker: Vida Dujmovic (University of Ottawa)

Title: Product structure theorem and clustered graph colourings

Abstract: Using a recent result on product structure of apex minor free graphs,
we give simple proofs on clustered colourings of such graphs, more
specifically those that exclude K_{s,t} as a subgraph.

Speaker: Stephen Hartke (University of Colorado Denver)

Title: Subcubic planar graphs are 7-square-choosable

Abstract: The square of a graph G is the graph with vertex set V(G) where two vertices are adjacent if their distance in G is at most 2. Wegner (1977) conjectured that the square of any subcubic planar graph is 7-colorable, which was proved independently by Thomassen (2018) and Hartke, Jahanbekam, and Thomas (2019+). Cranston and Kim (2008) asked whether the list chromatic number of the square of any subcubic planar graph is also at most 7. We answer this question in the affirmative, proving that all subcubic planar graphs are 7-square-choosable.

This is joint work with Luke Nelsen.

Speaker: Chính T. Hoàng (Wilfrid Laurier University)

Title: Coloring graphs with forbidden induced subgraphs

Abstract: Let F4 be a set of four-vertex graphs. For any set F4, it is known that COLORING F4-free graphs is NP-hard or solvable in polynomial time, except when F4 is one of the following three sets: {claw, 4K1}, {claw, 4K1, co-diamond}, {4K1, C4}. In this talk, we survey recent advances on these three open problems. We will discuss the two tools that have been proved to be useful in attacking this kind of problems: perfect graph theory, and the theory of clique width.

Speaker: Ringi Kim (KAIST)

Title: Decomposition of a planar graph into a forest and a 3-choosable subgraph

Abstract: Recently, Grytczuk and Zhu proved that every planar graph G contains a matching M such that G-M is 4-choosable. In this talk, we show that every planar graph G contains a forest F such that G-E(F) is 3-choosable. We also show that a forest cannot be replaced by a subgraph of maximum degree at most 3 or a star forest.

This is joint work with Seog-Jin Kim and Xuding Zhu.

Speaker: A. Kostochka (University of Illinois at Urbana-Champaign)

Title: Defective DP-colorings of sparse graphs and multigraphs

Abstract: We introduce and study defective DP-colorings of graphs and multigraphs, concentrating on colorings with two colors. In this setting, for each vertex of a (multi)graph G, the list $L(v)=\{p(v),r(v)\}$ has two colors, and we say that G is DP-(i,j)-colorable if for every 2-fold cover $H(G)$ of G, one can choose a color $\phi(v)\in L(v)$ for each vertex v so that if $\phi(v)=p(v)$ (respectively, $\phi(v)=r(v)$), then $\phi(v)$ is adjacent to at most i (respectively, j) other chosen colors.

We prove exact lower bounds on the number of edges in DP-(i,j)-critical n-vertex multigraphs for all pairs (i,j), and in simple DP-(i,j)-critical n-vertex graphs --- for $(i,j)\in \{(1,1),(2,2)\}$. For some pairs (i,j), our bounds for defective DP-colorings are better than known bounds for ordinary defective colorings.

This is joint work with Yifan Jing, Fuhong Ma, Pongpat Sittitrai and Jingwei Xu.

Speaker: Richard Lang (University of Waterloo)

Title: Asymptotically good local list edge colourings

Abstract: We study list edge colourings under local conditions. Our main result is an analogue of Kahn's theorem in this setting. More precisely, we show that, for a graph G with sufficiently large maximum degree Delta and minimum degree delta > log^25 Delta, the following holds. Suppose that lists of colours L(e) are assigned to the edges of G, such that, for each edge e=uv,

|L(e)| > (1+o(1)) max{deg(u),deg(v)}.

Then there is an L-edge-colouring of G. We also provide extensions of this result for hypergraphs and correspondence colourings, a generalization of list colouring.

Joint work with Marthe Bonamy, Michelle Delcourt, and Luke Postle.

Speaker: Bernard Lidicky (Iowa State University)

Title: Coloring count cones of planar graphs

Abstract: For a plane near-triangulation G with the outer face bounded by a cycle C, let $n^\star_G$ denote the function that to each 4-coloring ψ of C assigns the number of ways ψ extends to a 4-coloring of G. The block-count reducibility argument (which has been developed in connection with attempted proofs of the Four Color Theorem) is equivalent to the statement that the function $n^\star_G$ belongs to a certain cone in the space of all functions from 4-colorings of C to real numbers. We investigate the properties of this cone for $|C|=5$, formulate a conjecture strengthening the Four Color Theorem, and present evidence supporting this conjecture.

This is a joint work with Zdeněk Dvořák.

Speaker: Jie Ma (University of Science and Technology of China)

Title: Counting critical subgraphs in k-critical graphs

Abstract: A graph is k-critical if its chromatic number is k but any its proper subgraph has chromatic number less than k. Let $k\geq 4$. Gallai asked in 1984 if any k-critical graph on n vertices contains at least n distinct $(k-1)$-critical subgraphs. Improving a result of Stiebitz, Abbott and Zhou proved in 1995 that every such graph contains $\Omega(n^{1/(k-1)})$ distinct $(k-1)$-critical subgraphs. Since then no progress had been made until very recently, Hare resolved the case $k=4$ by showing that any 4-critical graph on n vertices contains at least $(8n-29)/3$ odd cycles. We mainly focus on 4-critical graphs and develop some novel tools for counting cycles of specified parity. Our main result shows that any 4-critical graph on n vertices contains $\Omega(n^2)$ odd cycles, which is tight up to a constant factor by infinite many graphs. As a crucial step, we prove the same bound for 3-connected non-bipartite graphs, which may be of independent interest. Using the tools, we also give a very short proof to the Gallai's problem for the case $k=4$. Moreover, we improve the longstanding lower bound of Abbott and Zhou to $\Omega(n^{1/(k-2)})$ for the general case $k\geq 5$.

Joint work with Tianchi Yang.

Speaker: Owen Merkel (University of Waterloo)

Title: An optimal χ-bound for (P_6, diamond)-free graphs

Abstract: Given two graphs H_1 and H_2, a graph is (H_1, H_2)-free if it contains no induced subgraph isomorphic to H_1 or H_2. Let P_t be the path on t vertices and K_t be the complete graph on t vertices. The diamond is the graph obtained from K_4 by removing an edge. We show that every (P_6, diamond)-free graph G satisfies $\chi(G) \le \omega(G)+3$, where $\chi(G)$ and $\omega(G)$ are the chromatic number and clique number of G, respectively. This bound is attained by the complement of the 27-vertex Schl\"afli graph.

This is joint work with Kathie Cameron and Shenwei Huang.

Speaker: Bojan Mohar (Simon Frasier University)

Title: The Last Temptation of William T. Tutte

Abstract: In 1999, at one of his last public lectures, Tutte discussed a question he had considered since the times of the Four Color Conjecture.

He asked whether the 4-coloring complex of a planar triangulation could have two components in which all colorings had the same parity. The talk will start by outlining the basics about coloring complexes, and end by answering Tutte's question to the contrary of his speculations by showing that there are triangulations of the plane whose coloring complexes have arbitrarily many even and odd components.

This is joint work with Nathan Singer.

Speaker: Ben Moore (University of Waterloo)

Title: Decomposing sparse graphs into pseudoforests

Abstract: A classical theorem of Hakimi states that a graph G decomposes into k pseudoforests if and only if the maximum average degree of G is at most 2k. We prove a strengthening of this theorem. For any integers k and d, if the maximum average degree of G is at most 2k + 2d/(k+d+1), then G decomposes into k+1 pseudoforests, such that one of the pseudoforests has each connected component having at most d edges.

Speaker: Francois Pirot (Radboud University)

Title: Using hard-core distributions for sparse graph colouring

Abstract: Attached (too long, tex heavy to copy)

Speaker: Pawel Pralat (Ryerson University)

Title: Clique coloring of binomial random graphs

Abstract: A clique colouring of a graph is a colouring of the vertices so that no maximal clique is monochromatic (ignoring isolated vertices). The smallest number of colours in such a colouring is the clique chromatic number. In this paper, we study the asymptotic behaviour of the clique chromatic number of the random graph G(n, p) for a wide range of edge-probabilities p = p(n). We see that the typical clique chromatic number, as a function of the average degree, forms an intriguing step function.

This is joint work with Colin Mcdiarmid and Dieter Mitsche.

Speaker: Greg Puleo (Auburn University)

Title: Some special cases of the strong coloring conjecture

Abstract: Haxell proved that if H is a graph of maximum degree at most Δ which is augmented by adding any number of vertex-disjoint copies of $K_{2\Delta}$, then the resulting graph G has an independent set hitting each of the added cliques. The "strong coloring conjecture" states that, under these conditions, a much stronger conclusion holds, namely that $\chi(G) = 2\Delta$. In the case where the added cliques partition H, we are asking not just for a single independent set that hits all the cliques, but for a partition of the vertex set into such independent sets. When $\Delta = 1$ this is just the statement that every 2-edge-colorable graph is bipartite, but even for $\Delta = 2$ the problem has stood open since at least the early 2000s. We will discuss some recent partial results on the $\Delta=2$ case of the conjecture.

This is joint work with Jessica McDonald.

Speaker: Matej Stehlik (Université Grenoble Alpes)

Title: Edge-critical subgraphs of Schrijver graphs

Abstract: Let n, k be integers such that $k \geq 1$ and $n \geq 2k$. The vertices of the Kneser graph $KG(n, k)$ are all k-subsets of $\{1, . . . , n\}$, with two subsets joined by an edge if and only if they are disjoint. Lovász [2] showed that the chromatic number of $KG(n, k)$ is $n-2k+ 2$, thereby proving a long-standing conjecture of Kneser [1]. Shortly thereafter, Schrijver [3] sharpened Lovász's result by showing that k-subsets containing no consecutive elements, or the pair $\{1, n\}$, induce a subgraph $SG(n, k)$, nowadays known as the Schrijver graph, with the same chromatic number. Schrijver also showed that this is best-possible, in the sense that the chromatic number drops whenever a vertex of $SG(n, k)$ is removed; in other words, the graph $SG(n, k)$ is vertex-critical. There is also a stronger notion of criticality: a graph is said to be edge-critical if the chromatic number drops whenever an edge is removed. It can be shown that $SG(n, k)$ is not edge-critical, unless $k = 1$ or $n = 2k + 1$. Thus, it is desirable to define an edge-critical $(n-2k+2)$-chromatic subgraph $XG(n, k)$ of $SG(n, k)$. In this talk, we provide such a construction for the case $k = 2$. Note that the vertices of $SG(n, 2)$ can be identified with the $n(n - 3)/2$ diagonals of a convex n-gon with vertices $1, . . . , n$, where the edges are pairs of diagonals with disjoint endpoints. Define a spanning subgraph $XG(n, 2)$ of $SG(n, 2)$ as follows. The vertices of $XG(n, 2)$ are the diagonals of a convex n-gon with vertices $1, . . . , n$, and the are pairs of diagonals that either cross, or do not cross and the unique face incident to both diagonals is not incident to the vertex 1. We will show that the graph $XG(n, 2)$ is $(n - 2)$-chromatic and edge-critical.

References

[1] M. Kneser. Aufgabe 300. Jber. Deutsch. Math.-Verein. 58 (1955).
[2] L. Lovász. Kneser's conjecture, chromatic number, and homotopy. J. Combin. Theory, Ser. A 25 (1978), 319–324.

[3] A. Schrijver. Vertex-critical subgraphs of Kneser graphs. Nieuw Arch. Wisk. (3) 26(3) (1978), 454–461.

Speaker: Matt Yancey (Institute for Defense Analyses)

Title: The exceptional sparse near-bipartite graphs.

Abstract: If the vertex set of a graph can be partitioned into a forest and an independent set, then we say it is near-bipartite. We recently showed that if every vertex set W of simple graph G that contains $e(W)$ edges satisfies that $8|W| - 5e(w) > -5$, then either G is near-bipartite or contains a subgraph isomorphic to a graph in a finite family of graphs, $\mathcal{H}$. The family $\mathcal{H}$ is large, with over 100 members, and is described recursively. In this talk, we will establish the finite-ness of $\mathcal{H}$ by studying the relationship between near-bipartite graphs and 3-colorable graphs.

This is joint work with Dan Cranston.

Speaker: Dantong Zhu (Georgia Tech)

Title: The Extremal Function for K10 Minors.

Abstract: For positive integers t and n, the maximum number of edges that an n-vertex graph with no Kt minor can have is known as the extremal function for Kt minors. In 1968, Mader proved that for every integer t = 1, 2, ..., 7, a graph on n ≥ t vertices and at least (t−2)n− t−1 2 + 1 edges has a Kt minor. Jørgensen showed that a graph on n ≥ 8 vertices and at least 6n − 20 edges either has a K8 minor or is isomorphic to a graph obtained from disjoint copies of K2,2,2,2,2 by identifying cliques of size 5. Song and Thomas further generalized the results for K9 minors. The extremal functions for Kt minors where t ≤ 9 have been important for proving several results related to Hadwiger's conjecture. In this talk, I will discuss our work-in-progress on the extremal function for K10 minors.

This is joint work with Robin Thomas.