The Typical Structure of Intersecting Families

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1. Preliminaries

- Motivation
- Definitions
- Hypergraphs

Extremal Combinatorics

- How large can a system be under certain restrictions?
- Can we enumerate such systems?
- Can we describe the "typical structure"?

Theorem (Mantel 1907)

The maximum number of edges in an n vertex triangle-free graph is $\left\lfloor \frac{n^2}{4} \right\rfloor$.

Theorem (Erdős–Kleitman–Rothschild 1976)

There are at most $2^{(1+o(1))\cdot ex(n,K_t)}$ K_t-free graphs on n vertices.

Sharp sparse analogue

Corollary (Balogh–Morris–Samotij–Warnke 2013+)

Almost every K_t-free n vertex graph with $m > m(n)$ edges is $(t-1)$-partite, where $m(n)$ is best possible.

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- k-uniform hypergraph on $[n]$, $\mathcal{F} \subseteq \binom{[n]}{k}$ is **intersecting** if $\forall F, G \in E(\mathcal{F})$, $|F \cap G| \geq 1$
- $\mathcal{F} \subseteq \binom{[n]}{k}$ is **trivial** if each edge contains a fixed vertex

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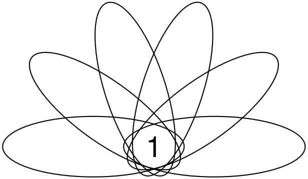

How large can an intersecting family be?

Theorem (Erdős–Ko–Rado 1961)

If $n \geq 2k$ and $\mathcal{F} \subseteq \binom{[n]}{k}$ is intersecting, then

$$e(\mathcal{F}) \leq \binom{n-1}{k-1}.$$

For $n > 2k$ we have equality only if $\mathcal{F}$ is trivial.

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- Let $\mathcal{H}^k(n,p)$ be the k-uniform hypergraph on $[n]$ with each edge included independently with probability p
- Let $i(\mathcal{H}) := \max\{e(\mathcal{F}) : \mathcal{F} \subseteq \mathcal{H}, \mathcal{F} \text{ intersecting}\}$

Theorem (Erdős–Ko–Rado Restated)

If $n \geq 2k$, then

$$i\left(\mathcal{H}^k(n,1)\right) = i\left(\binom{[n]}{k}\right) \leq \binom{n-1}{k-1}.$$

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$\mathcal{H}$ **satisfies Erdős-Ko-Rado** if $i(\mathcal{H})$ is achieved only by a trivial subhypergraph

Question (Balogh–Bohman–Mubayi 2009)

Given $2 \leq k < n/2$, for which functions $p = p(n) \in [0, 1]$ does $\mathcal{H}^k(n, p)$ satisfy Erdős-Ko-Rado?

Balogh–Bohman–Mubayi settled this for $k < n^{1/2-o(1)}$

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Theorem (Gauy–Hàn–Oliveira 2014+)

Let $\delta = \delta(n) > 0$. If $\ln n \ll k < \frac{(1-\delta)n}{2}$ and $p \gg \frac{1}{\delta}(\frac{\ln n}{k})^{1/2}$, then almost surely as $n \to \infty$

$$i(\mathcal{H}^k(n,p)) = (1 + o(1))p\binom{n-1}{k-1}.$$

Trivial intersecting subfamilies are asymptotically largest

Things change around $k = \sqrt{n}$.

- For smaller k, two random k-sets are typically disjoint
- For larger k, the opposite holds

Result (Hamm–Kahn 2014+)

For any fixed $c < 1/4$, and

$$k < \sqrt{cn \log n}$$

characterize $p = p(n, k)$ for which then $\mathcal{H}^k(n, p)$ satisfies Erdős–Ko–Rado w.h.p.

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There is a fixed $\varepsilon > 0$ such that the following holds. If $n = 2k + 1$ and $p > 1 - \varepsilon$, then $\mathcal{H}^k(n,p)$ satisfies Erdős–Ko–Rado w.h.p.

Theorem (Balogh–Das–D.–Liu–Sharifzadeh 2015)

For $3 \leq k \leq \frac{n}{4}$, if

$$p = p(n) \geq c \cdot n \log\left(\frac{ne}{k}\right) \frac{\binom{2k}{k}\binom{n}{k}}{\binom{n-k}{k}^2},$$

then w.h.p. every largest intersecting subhypergraph of $\mathcal{H}^k(n,p)$ is trivial.

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2. Intersecting Permutations

- Background
- Main Result

- Let S_n denote the **symmetric group** on $[n]$
- $\mathcal{F} \subseteq S_n$ is **intersecting** if $\forall\, \sigma, \pi \in \mathcal{F}$

$$|\sigma \cap \pi| := |\{i \in [n] : \sigma(i) = \pi(i)\}| \geq 1$$

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for fixed $i, j \in [n]$,

$$|\{\sigma \in S_n : \sigma(i) = j\}| = (n-1)!$$

$$(4-1)! = 3! = 6$$

Theorem (Frankl–Deza 1977)

If $\mathcal{F} \subseteq S_n$ is intersecting, then $|\mathcal{F}| \leq (n-1)!$.

Theorem (Cameron–Ku / Larose–Malvenuto 2003)

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How large can a non-trivial, intersecting family be?

Theorem (Ellis 2008)

For n sufficiently large, the largest non-trivial intersecting $\mathcal{F} \subseteq S_n$ have size

$$\left(1 - \frac{1}{e} + o(1)\right)(n-1)!.$$

$\{\sigma \in S_n : \sigma(1) = 1, \sigma(j) = j \text{ for some } j > 2\} \cup \{(1\ 2)\}$

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Theorem (Balogh–Das–D.–Liu–Sharifzadeh 2015)

1. *The number of intersecting families of permutations is*

$$(n^2 + o(1))2^{(n-1)!}.$$

2. *Almost every intersecting family of permutations is trivial.*

We will use the skew-symmetric Bollobás set-pairs inequality

Theorem (Frankl 1982)

Let $A_1, \ldots, A_m$ be sets of size a and $B_1, \ldots, B_m$ be sets of size b such that $A_i \cap B_i = \emptyset$ and $A_i \cap B_j \neq \emptyset$ for every $1 \leq i < j \leq m$. Then $m \leq \binom{a+b}{a}$.

Proposition (Balogh–Das–D.–Liu–Sharifzadeh 2015)

The number of maximal intersecting families in S_n is at most

$$\sum_{i=0}^{\frac{1}{2}\binom{2n-1}{n}} \binom{n!}{i} < n^{n2^{2n-1}}.$$

Proof.

- Pick $\mathcal{F}$ max int, $\mathcal{I}(\mathcal{F}) := \{\pi \in S_n : \forall \sigma \in \mathcal{F},\ |\pi \cap \sigma| \geq 1\}$
- Let $\mathcal{F}_0 = \{\sigma_1, \ldots, \sigma_s\} \subset \mathcal{F}$ be a minimal generating set $(\mathcal{F} = \mathcal{I}(\mathcal{F}_0))$
- $\forall i \in [s]$, $\exists \tau_i \in S_n$ such that $|\sigma_j \cap \tau_i| = 0$ iff $i = j$
- Assign $H_\pi = \{(1, \pi(1)), \ldots, (n, \pi(n))\}$

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  for $1 \leq i \leq s$, $A_i = H_{\sigma_i}$ and $B_i = H_{\tau_i}$, and
  for $s+1 \leq i \leq 2s$, $A_i = H_{\tau_{i-s}}$ and $B_i = H_{\sigma_{i-s}}$
- Then $s \leq \frac{1}{2}\binom{2n-1}{n}$

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Proof.

- To every maximal family $\mathcal{F}$ we may assign a distinct generating set of size at most $\frac{1}{2}\binom{2n-1}{n}$
- The upper bound follows since $n! \leq n^n$ and $\binom{2n-1}{n} \leq 2^{2n-1}$

□

The general method

N_0 size of the largest trivial intersecting family
N_1 size of the largest non-trivial intersecting family
M upper bound on the number of maximal families

Observation

Any subset of a trivial intersecting family is itself trivial.

There are at least 2^{N_0} trivial families

Observation

Any non-trivial intersecting family must be contained inside a maximal non-trivial intersecting family.

There are at most $M2^{N_1}$ non-trivial families

$$\frac{M2^{N_1}}{2^{N_0}} \to 0$$

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Thank you for listening!