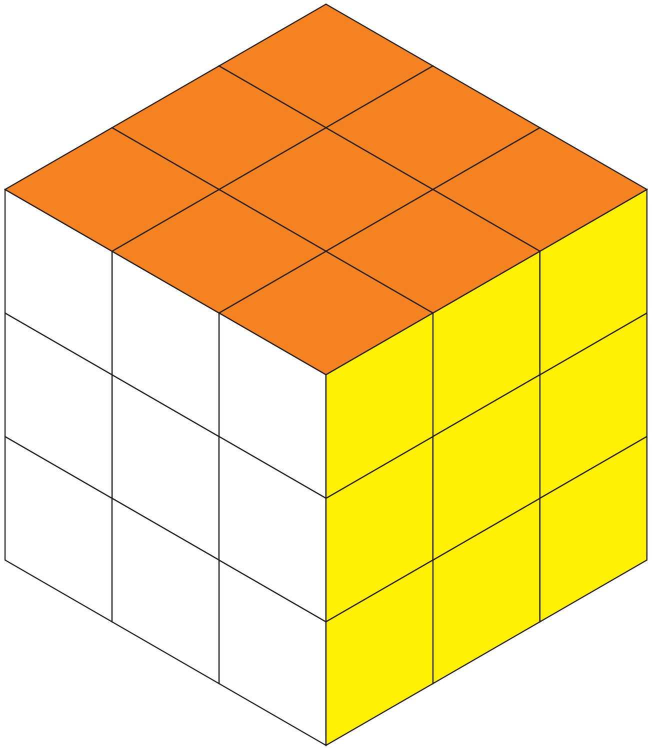
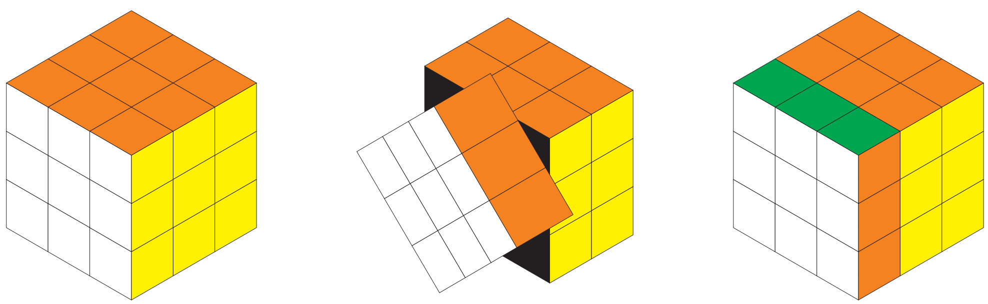
# The Rubik's Cube



# The Moves

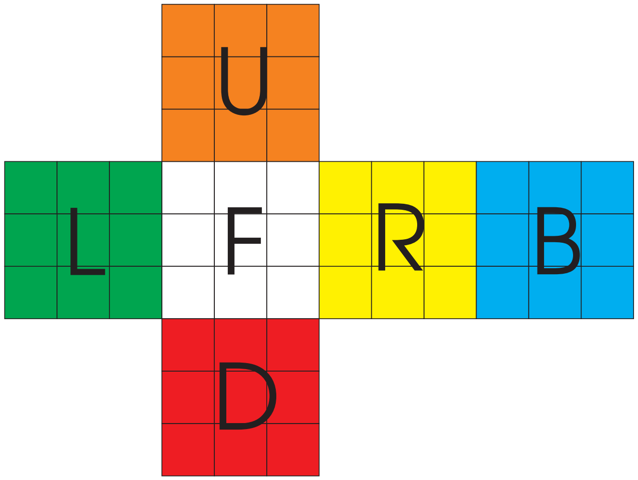
Each face of the cube can be rotated.

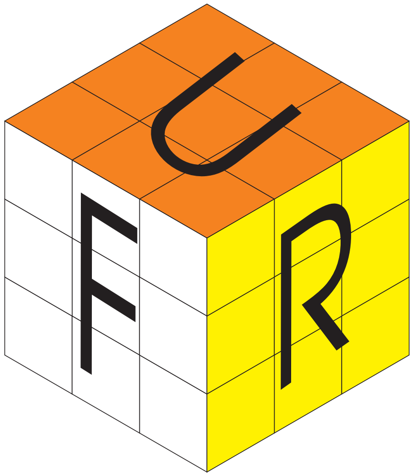


The result is a permutation of the stickers and physical pieces (cubies) that make up the cube.

## The Puzzle

The faces of the cube are denoted:
(F)ront, (B)ack, (L)eft, (R)ight, (U)p, and (D)own




The cube group, $G_{cube}$, is the permutation group generated by the actions of the six face turns on the stickers.

# Only Five Generators Required

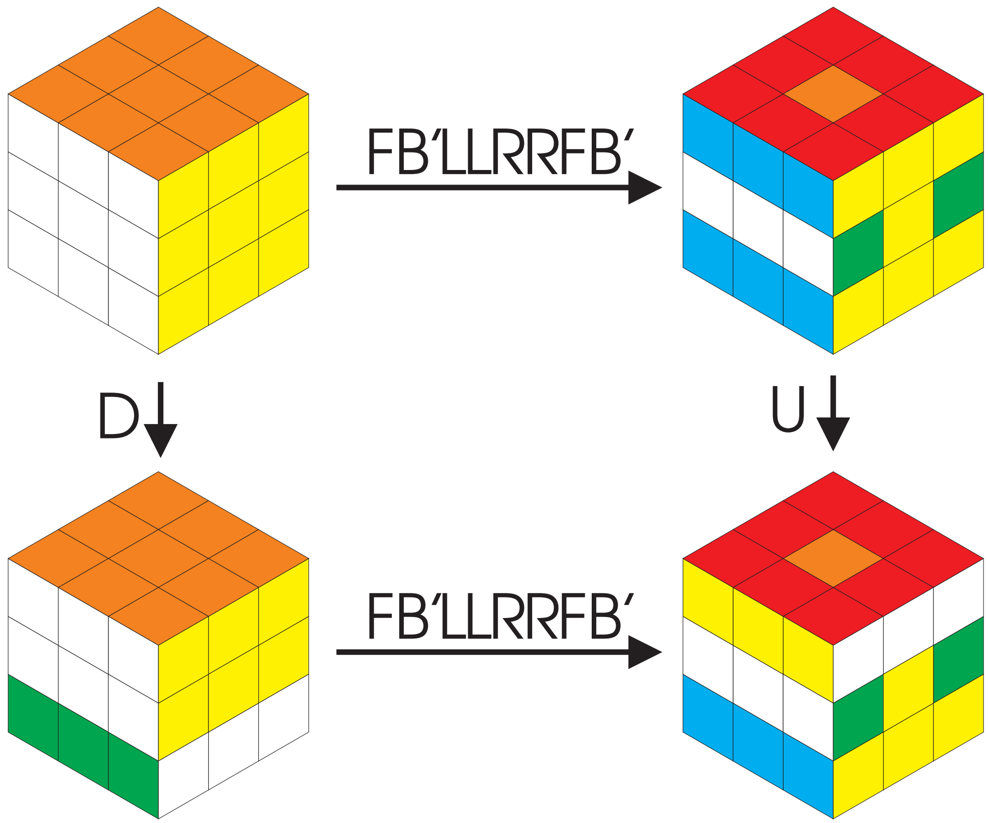



Figure 1: A commuting diagram

# Counting the States

We consider first a larger permutation group, $G$ of all permutations obtainable by taking apart the cube and putting it back together.

$$G \simeq G_{corner} \oplus G_{edge}$$

$G_{corner}$ and $G_{edge}$ are wreath products.

$$G_{corner} \simeq S_8[A_3] \qquad G_{edge} \simeq S_{12}[S_2]$$

The order of $G$ is thus:

$$|G| = |G_{corner}| \cdot |G_{edge}| = 8! \times 3^8 \times 12! \times 2^{12}$$

| | -1 | 0 | +1 |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

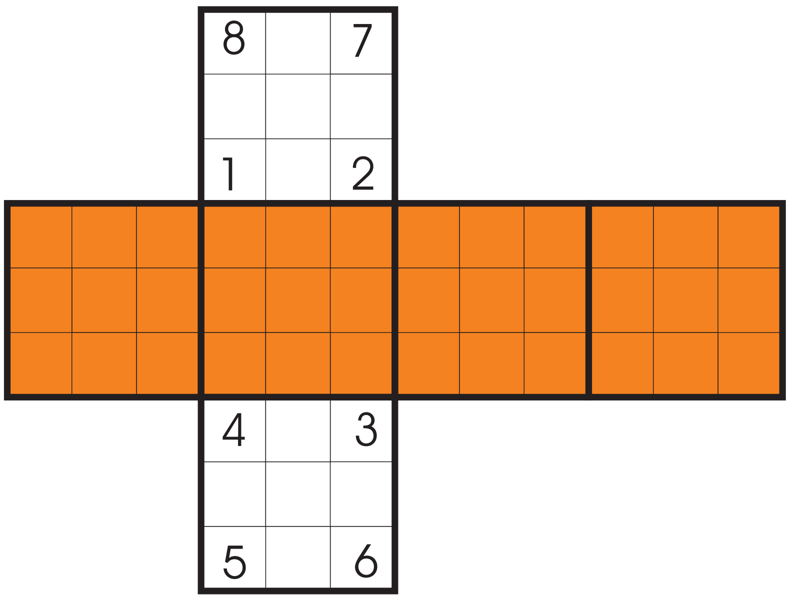
## The Order of $G_{cube}$

We show that $G_{cube}$ has index 12 in $G$ and thus:

$$|G_{cube}| = 12! \times 8! \times 2^{10} \times 3^7$$

- 18 cubie positions determine the remaining 2
- 11 edges orientations determine the twelfth
- 7 corner orientations determine the eighth

# An Alternate Colouring



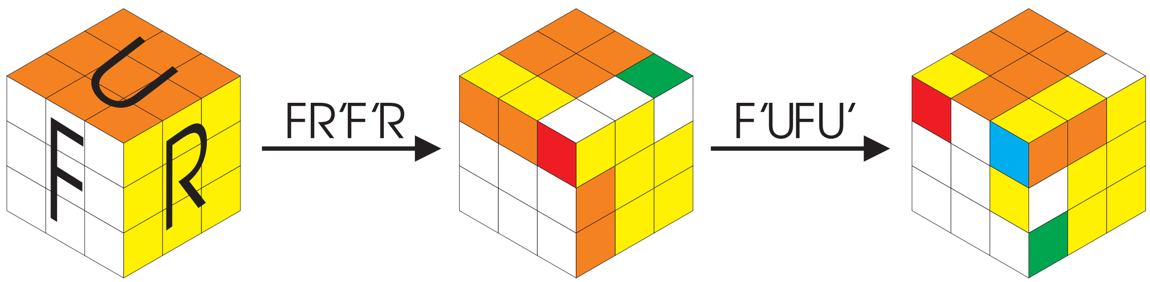

| | -1 | 0 | +1 |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

| | -1 | 0 | +1 |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

| | -1 | 0 | +1 |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

The sum of the orientations of the corners is always zero.

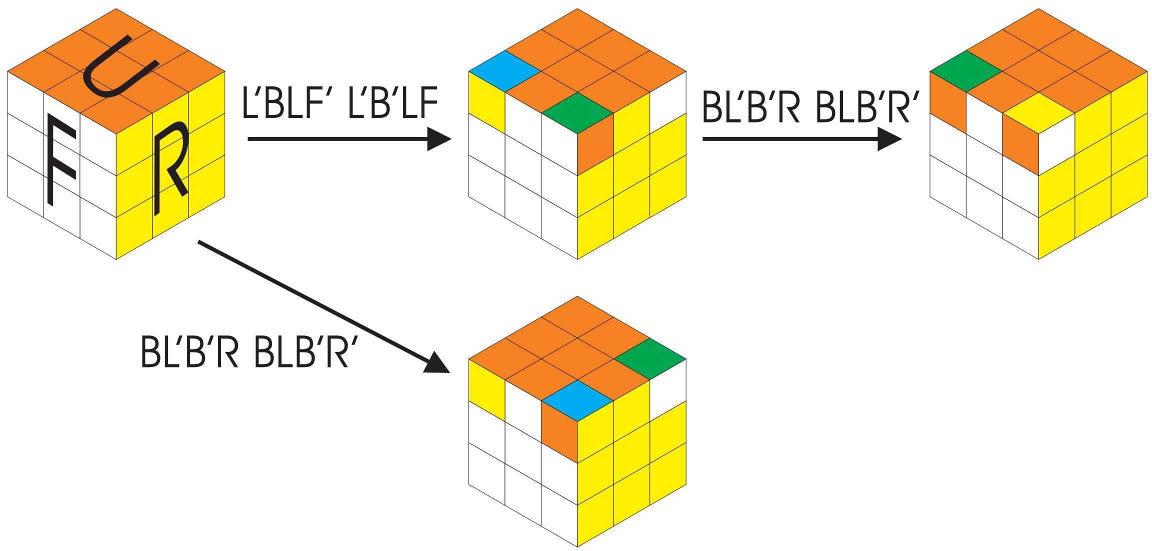
# Generating the Edge Group




Two related commutators.


The restriction to the edge group.

# Generating the Corner Group




Another commutator gives us a three cycle to position corners. Combining this with a related commutator lets us orient the corners.

# Diameter of the Cayley graph

For the quarter turn metric. An edge when states differ by an element of $\{L, R, F, B, U, D, L', R', F', B', U', D'\}$.

- Lower bound of 24 for the super-flip

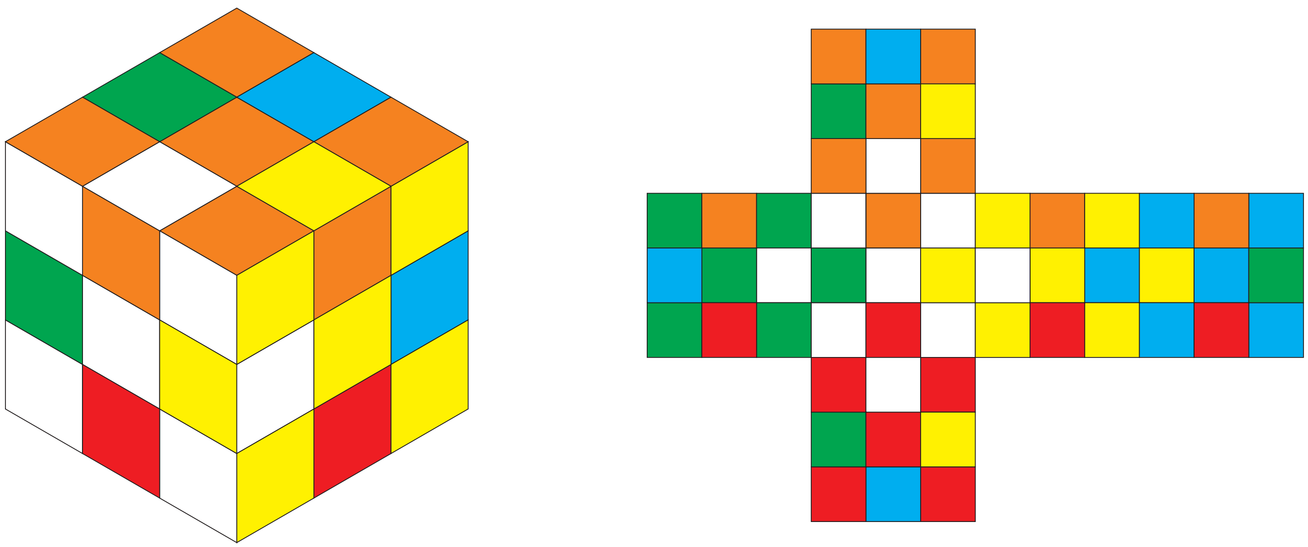


Figure 2: $R'U^2BL'FU'BDFUD'LD^2F'RB'DF'U'B'UD'$

- Upper bound of 42 using Kloosterman's modification of Thistlethwaite's algorithm.

$$G_0 = < L, R, F, B, U, D >$$

$$G_1 = < L, R, F, B, U^2, D^2 >$$

$$G_2 = < L, R, F^2, B^2, U^2, D^2 >$$

$$G_3 = < L^2, R^2, F^2, B^2, U^2, D^2 >$$

See http://web.idirect.com/ cubeman/dotcs.txt for the face turn metric.

http://www.geocities.com/jaapsch/puzzles/cayley.htm puts the state

$$U^2 D^2 L F^2 U' D R^2 B U' D' R L F^2 R U D' R' L U F' B'$$

at distance 26q.