Introduction

The process of generating valid inequalities for a mixed integer problem

\[ P = \min \{ f(x) : Ax = b, \ x \geq 0, \ x \in \mathbb{Z}, \ \gamma \in \mathcal{J} \} \]

generally involves first considering a relaxation of the problem by dropping some of the constraints (integrality, non-negativity, or general linear constraints). Then, cuts can be computed for the relaxed problem. Therefore, by examining how tight the relaxed problem is, we get what is attainable in the best case with cuts based on that relaxation.

In practice, we measured the gap closed by five relaxations in the following way:

\[ \%gc = 100 \frac{z_{gap} - z_{opt}}{z_{opt}} \]

on most problems of the in the mip10l [5] and mip8l03 [5].

The extent of this information has three important limitations:

- We do not know the gap actually closed by the cuts: cutting-plane algorithms may or may not converge to the underlying relaxation.
- All the relaxations we consider are based on the status of the variables at the LP optimum. Therefore, our measurements only regard independent inequalities added after the root node of the MIP.
- Due to the ease of the objective function as an indicator, when the relaxation deals only with part of the roots of the problem, we still need to consider the others. This in order to keep the basis matrix full-rank in the original problem, and have defined values for all our variables. An alternative would be to compute all the facets of the relaxation that are binding at its optimal value, and add them to the MIP. But that would not be conceivable for large-scale problems.

However, this provides us with upper-bounds on the gap closure obtained using cuts derived from each relaxation. We thus have a partial indication of the usefulness of each type of cut for the problem we study.

In this approach, we can obtain a generalization of the work of Faschetti and Monaci [1] on the group and corner relaxations and part of our experiments overlaps with theirs (specifically, on the group relaxation), with similar outcomes.

Relaxations

The group relaxation [13][14][16][17][18][19][20] consists in dropping non-negativity constraints on all basic variables, i.e. given B and T the index set of respectively basic and nonbasic variables in the optimal solution of \( P \sqrt{SP} \).

\[ P_{gap} = \min \{ z_{opt} : Ax = b, \ x \geq 0, \ x \in \mathbb{Z}, \ \gamma \in \mathcal{J} \} \]

The mixed integer set \( P_T \) presented in [6] and [8], is related to the one suggested in [5] for deriving inequalities from two rows of the simplex tableau:

\[ P_T = \min \{ z_{opt} : Ax = b, \ x \geq 0, \ x \in \mathbb{Z}, \ \gamma \in \mathcal{J} (\cap B) \} \]

The set \( P_T \)-lifting adds back to \( P_T \) the integrality constraints on the non-basic variables [9][7].

The relaxation \( \text{“} b \text{-bounded} P_T \) adds lower and upper bounds (when available in the original problem) on the basic variables of \( P_T \) [10][12][11], while \( \text{“} b \text{-bounded} P_T \) considers bounds on the non-basic variables [2].

On the 47 problems for which we could solve to optimality both the group and \( P_T \) relaxations, we have the following statistics:

\[
\begin{align*}
\text{Average (\%gc)} & : 34.69 \quad 26.48 \\
\text{Standard deviation (\%gc)} & : 34.29 \quad 42.35 \\
\end{align*}
\]

And they are distributed in the function of the gap-closure as illustrated on the right.

Conclusions

As explained before, the average gap closures presented here must be taken with caution: they are not necessarily directly linked to efficiency in the corresponding cuts. However, an interesting fact might be the apparent non-correlation, given one problem, in the gap closed by the group and \( P_T \) relaxations (see graph below). This can lead us to think that the per-problem usefulness of a given type of cut might be highly determined by the tightness of the underlying relaxation.

Further developments

Some refinements can be made in the handling of models:

- The additional bounds on variables could be evaluated by optimizing over the LP when they are not present in the initial formulation.
- The slack variables could be made integer in pure-integer constraints. However, this leads to dramatically higher computing times.
- More complete tables will be available with extended memory and CPU time limits (currently 3GB and 26).

References


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