Sample Exam
PMath 360, 2005

1. **Homogeneous coordinates**
   (a) Three lines are given with coordinates $[1,1,1]$, $[2,3,a]$, and $[5, a+6, a+3]$.
   For which values of a are the three lines coincident?
   (b) Given three distinct lines $L : [l_1,\ l_2,\ l_3]$, $M : [m_1,\ m_2,\ m_3]$ and $N : [n_1,\ n_2,\ n_3]$, prove that L, M and N meet at some point P if and only if the coordinate vectors satisfy an equation of the form
   $$c_1[l_1,\ l_2,\ l_3] + c_2[m_1,\ m_2,\ m_3] + c_3[n_1,\ n_2,\ n_3] = 0$$

2. **Matrix of a conic**
   Three points are given by $A : (1,0,0)$, $B : (0,0,1)$ and $C : (1,1,1)$.
   Two lines are given by $T1 : [0,2,5]$ and $T2 : [5,4,0]$.
   Find the matrix of the conic that satisfies all three of these conditions:
   (a) $T1$ is tangent to the conic at A
   (b) $T2$ is tangent to the conic at B
   (c) The point C is on the conic.

3. **Tangents to a conic**
   A conic is given by the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.
   (a) Verify that for all t, the point $T : (1,\ -t^2,\ t)$ is on the conic.
   (b) The point $B : (b,\ 0,\ 1)$ is outside (exterior to) the conic.
   For what values of t is the line BT tangent to the conic?

4. **Inversion, Orthogonal circle**
   A circle Σ and a point P are given.

   (a) List the construction steps for the inverse of P with respect to Σ. The construction is to be one that works for P inside or outside or on Σ. Include a labeled figure that shows your steps.

   (b) Given that the point P is outside Σ, give a construction for the circle whose centre is P and is orthogonal to Σ. Again list your steps and show a sketch.

5. **Hermitian Matrices and Circles**

   (a) Prove that the three circles represented by
   $H_1 = \begin{bmatrix} 1 & 0 \\ 0 & -100 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 & -\overline{\gamma} \\ -\gamma & 0 \end{bmatrix}$ and $H_3 = \begin{bmatrix} 0 & \overline{\gamma} \\ \gamma & -100 \end{bmatrix}$
   lie in a common pencil, regardless of the value of γ.

   (b) Find the cosine of the angle determined by H_1 and H_2.

   (c) Classify the pencil formed by H_1 and H_2.

6. **Stereographic Projection**
   Let $\mathcal{S}$ be the sphere with centre $(0,\ 0,\ 0)$ and radius 1. Let $N:\ (0,\ 0,\ 1)$.

   (a) Given $P:\ (u,\ v,\ w) \in\ \mathcal{S}$, with $w \neq 1$, find $Q:\ (x,\ y,\ 0)$ so that P, Q and N are collinear.

   (b) Given $R:\ (x,\ y,\ 0)$, find $T:\ (u,\ v,\ w) \in \mathcal{S}$, so that R, T and N are collinear.

   (c) Show that the plane $3x + 4y + 5z = 1$ meets the sphere $\mathcal{S}$ in a real circle.

   (d) Find the Hermitian matrix of the circle in the complex plane that is the projection of the circle that is the intersection of $\mathcal{S}$ with the plane given in (c).