Q#1 Cross Ratios

(a) Four collinear points A, B, C, D are given so that $(A, B; C, D) = 3/7$. Find the value of $(A, C; D, B)$.

(b) Let $U : (1, 0, 1)$, $V : (3, 0, 1)$, $W : (8, 0, 1)$, and $X : (x, 0, 1)$. Find the number x so that the cross ratio $(U, V; W, X)$ is $4/5$.

Q#2 Equation of conic

A certain conic is known to be given by a matrix M of the form

$$M = \begin{bmatrix} 0 & h & g \\ h & 0 & f \\ g & f & 0 \end{bmatrix}$$

Find the matrix of this conic if this additional condition is imposed: the polar of $(4, 2, 6)$ is $[3, 6, 2]$.

Q#3 Frame of reference:

(a) Verify that the four points $(1, 1, 1), (1, 2, 0), (1, 0, 0)$, and $(4, 8, 2)$ form a frame of reference for the projective plane.

(b) Find the matrix of the collineation that maps the four points of the standard frame $(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)$ to the frame given above.

Q#4 Stereographic projection.
In this equation, we use stereographic projection as defined in lecture, using S, the sphere with equation $u^2 + v^2 + w^2 = 1$, identifying the points $x + iy$ of the complex plane with points $(x, y, 0)$ in three space, and using $N : (0, 0, 1)$ as the centre of projection.

(a) Verify that the plane whose equation is $2u + 3v + 4w = 1$ meets S.

(b) Find the circle in the complex plane that corresponds to the above plane.

Q#5 Let Γ and Δ be two non-intersecting circles with distinct centres. Let P be any point not on the line of centres, not on Γ, and not on Δ.

(a) Show how to find a circle through P that is orthogonal to both Γ and Δ.

(b) Let C be any circle that is orthogonal to both Γ and Δ. Let ℓ be the line on the centres of Γ and Δ. Show that the two points of $\ell \bigcap C$ are inverses of each other with respect to Γ and simultaneously, with respect to Δ.

(c) Find a point Q that is the centre of a circle Σ such that the inverses of Γ and Δ (with respect to Σ) are concentric. Explain why you construction works.

Q#6 Three circles are given in the Euclidean plane. Each is tangent to the other two. Describe a step by step procedure to construct all the circles that are tangent to all three of the given circles.