PMath 360 Sample Final 2001

1. **Fundamental theorem**

   (a) Find the matrix of the collineation that maps the standard frame of reference to the frame $\{(1,1,1),(1,2,0),(1,0,1,),(6,7,4)\}$.

   (b) Using the answer to (a), find the images of the points $(1,-1,0)$ and $(0,-1,1)$ and the line $[1,1,1]$.

2. **Equation of conic**

   Find the equation of the conic that is

   tangent to $[0,1,0]$ at $(1,0,0)$,

   tangent to $[1,0,0]$ at $(0,1,0)$,

   and contains the point $(2,3,1)$.

3. **Theorem of Desargues**

   State the theorem of Desargues and show a sketch.

4. **Theorem of Pappus**

   State the theorem of Pappus and show a sketch.

5. **Hermitian Matrices**

   (a) Two circles in the complex plane are given by the Hermitian matrices $H_1 = \begin{bmatrix} 1 & 3-4i \\ 3+4i & 0 \end{bmatrix}$ and $H_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Find the matrix in the pencil determined by H_1 and H_2 that represents a line.

   (b) Find the Cartesian equations of the object represented by $\begin{bmatrix} 1 & -3-i \\ -3+i & -6 \end{bmatrix}$.

   (c) Do the same for the matrix $\begin{bmatrix} 0 & -2+i \\ -2-i & 7 \end{bmatrix}$.

6. **Inverses**

You are given a cirlce Σ with centre O and a point P distinct from O.

(a) Give the best procedure you know for constructing P^{Σ}.

(b) Suppose P is inside Σ. Give a procedure for finding a circle Γ so that $\Sigma = \Sigma^{\Gamma}$ and $O = P^{\Gamma}$.

7. **Orthogonal Circles**

Suppose Σ is a circle with centre O and P and Q are distinct points that are inverses of each other with respect to Σ. Prove that any circle through both P and Q is orthogonal to Σ.

8. **Stereographic Projection**

Let $\mathcal{S}$ be the sphere of radius 1 with centre at $(0, 0, 1)$. Let π be the plane with equation $z = 0$. [Note that $\mathcal{S}$ and π are tangent at $(0, 0, 0)$.]

Consider stereographic projection from the plane π to the sphere $\mathcal{S}$ from the point $N = (0, 0, 2)$ that maps points (x, y) of π points $(u, v, w,)$ on $\mathcal{S}$.

Find the equations for u, v, and w in terms of x and y.

9. **Tangent Circles**

Two cirlces C_1 and C_2 are given tangent to each other at P.

Give a procedure for finding a sequence of circles $D_1, D_2, D_3 \cdots$ each tangent to the next and all tangent to C_1 and C_2.

10. **Imaginary Circles**

Explain how to recognize an imaginary circle

(a) in terms of its Cartesian equation

(b) in terms of its Hermitian matrix

(c) in terms of its stereographic image.