1. Before you begin the test, fill in the information requested at the top of each page.
2. Show your work in the space provided. If more space is needed, continue your work on the back of the previous page, or on the blank page at the end. If you do, indicate (in the space provided) where your work is continued.
3. Highest marks will be given for work that is complete, concise, and clearly presented.

1. Inversion

Let C be any circle in the plane and let P be any point other than the centre of C.
(a) Give a construction for a point Q that is the inverse of P with respect to C.
(b) Prove that your construction works, namely that P and Q are inverses with respect to C.

2. Family of circles

Let $\mathcal{F}$ be the family of circles represented by these two matrices

$$H_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad H_2 = \begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}.$$

(a) What is the radius of the circle given by H_2 ?
(b) Find the matrix of the line in the family $\mathcal{F}$.
(c) Find the matrix of the circle in the family $\mathcal{F}$ that contains $1+2i$.

3. Family of circles 2

Let W be any circle and let P be any point not at the centre of W and not on W.

Let $\mathcal{S}$ be any circle with centre at P.
Let $Q = P^W$.
Let $\mathcal{F}$ be the family of all circles D on P and Q.
Let $\mathcal{F}^{\mathcal{S}}$ be the set of all the images of all circles in $\mathcal{F}$ under inversion with respect to $\mathcal{S}$.
Describe this family of images. (Justify your statements).

4. Stereographic projection

Let S = { (u, v, w) : $u^2 + v^2 + w^2 = 1.$ }, the unit sphere in three space with centre at the origin. Consider stereographic projection using the point N: (0, 0, 1), from S to the xy plane. As usual identify the complex point x+yi with the triple (x,y,0). You may assume that the map from S to the complex plane is given by

$$(u, v, w) \longrightarrow x+i\,y = (u+i\,v)/(1-w).$$

(a) Find the equation of the plane in three space that contains the stereographic image of the circle given by H = $\begin{bmatrix} 1 & i \\ -i & 7 \end{bmatrix}$.

(b) Let M be the plane with equation $3u + 4v - 5w = 1$. Find the Hermitian matrix representing the circle in the complex plane that is the stereographic image of the circle $S \cap M$.

5. Five points determine a conic

Find the matrix of the conic that contains the points
X: (1, 0, 0), Y: (0, 1, 0), Z: (0, 0, 1),
U: (1, 1, 1), and S: (1/29, 1/31, 1/37).

6. Collineation

Let four points in the Cartesian plane be given:
A: (0, 0), B: (1, 0), C: (9/10, 7/10), and D: (0, 8/10).
(a) Find the homogeneous coordinates of these two points $AB \cap CD$ and $BC \cap DA$.

(b) Find the 3 by 3 matrix of the collineation that maps the homogeneous coordinates of the four points of the unit square A, B, (1,1), and (0,1) to the homogeneous coordinates of A, B, C, and D, respectively. (Hint: Use the answers to (a).)