1. Inversion

Let C be any circle in the plane and let P be any point other than the centre of C.
(a) Give a construction for a point Q that is the inverse of P with respect to C.
(b) Prove that your construction works, namely that P and Q are inverses with respect to C.

2. Family of circles

Let $\mathcal{F}$ be the family of circles represented by the matrices

$$H1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } H2 = \begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}.$$

(a) What is the radius of the circle given by H2 ?
(b) Find the matrix of the line in $\mathcal{F}$.
(c) Find the matrix of the circle in $\mathcal{F}$ that contains $1+2i$.

3. Family of circles 2

Let $\mathcal{W}$ be any circle and let P be any point not on $\mathcal{W}$ and not at the centre of $\mathcal{W}$. Let Q be the inverse of P with respect to $\mathcal{W}$. Let $\mathcal{F}$ be the family of all circles on P and Q. Let Σ be any circle with centre at P. Consider the images of all circles in $\mathcal{F}$ under inversion with respect to Σ. Describe the family of images. (Justify your statements).

4. Stereographic projection

Let S = { (u, v, w) : $u^2 + v^2 + w^2 = 1$. }, the unit sphere in three space with centre at the origin. Consider stereographic projection using the point N: (0, 0, 1), from S to the xy plane. As usual identify the complex point $x+yi$ with the triple (x,y,0). You may assume that the map from S to the complex plane is given by

$$(u, v, w) \longrightarrow x+i\,y = (u+i\,v)/(1-w).$$

(a) Find the equation of the plane in three space that contains the stereographic image of the circle given by $H = \begin{bmatrix} 1 & i \\ -i & 7 \end{bmatrix}$.
(b) Let M be the plane with equation $3u + 4v - 5w = 1$. Find the Hermitian matrix representing the circle in the complex plane that is the stereographic image of the circle $S \cap M$.

5. Five points determine a conic

Find the matrix of the conic that contains the points
X: (1, 0, 0), Y: (0, 1, 0), Z: (0, 0, 1),
U: (1, 1, 1), and S: (1/29, 1/31, 1/37).

6. Collineation

Let four points in the Cartesian plane be given:

A: (0, 0), B: (1, 0), C: (9/10, 7/10), and D: (0, 8/10).

(a) Find the homogeneous coordinates of these two points: $AB \cap CD$ and $BC \cap DA$:
(b) Find the 3 by 3 matrix of the collineation that maps the homogeneous coordinates of the four points of the unit square A, B, (1,1), and (0,1) to the homogeneous coordinates of A, B, C, and D, respectively. (Hint: Use the answers to (a).)